ACCUMULATION OF FLUID FLOW ENERGY BY VIBRATIONS EXCITATION IN SYSTEM WITH TWO DEGREE OF FREEDOM

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Abstract. Very large achievements in mechanics of the field of aerodynamics and hydrodynamics exist. In the same time there are many technical tasks to be solved. One of the tasks group is find out new kinds of energy transformation from constant fluid flow to accumulators. Its needs optimization form and parameters of technical objects. A criteria like energy saving or its useful utilization from interaction with air or water flow may be used. In the first part of report a motion of a vibrator with constant air or water flow velocity excitation is optimized. The main task is to find out optimal control law for variation of area of vibrating object within limits. It is shown that optimal control action is on bounds of area limits. Synthesis of control law in phase coordinates shows that very effective real control law is change area of object when it stops right or left side. In the second part of report obtained optimal control law is realized in real vibration system.

Keywords: motion control, air or water flow excitation, energy utilization, optimal control, adaptive control, synthesis adaptive systems.

Introduction

Motion of a vibrator with two degree of freedom and constant air flow $\overline{V_0}$ excitation is investigated (Fig. 1). System consists of masses m_1, m_2 with springs c_1, c_{12} and dampers b_1, b_{12}. The main idea is to find out optimal control law in time for variation of additional area $S(t)$ of vibrating mass m_2 within limits (1):

$$S_1 \le S(t) \le S_2, \tag{1}$$

where S_1 – lower level of additional area of mass m_2;
S_2 – upper level of additional area of mass m_2;
t – time.

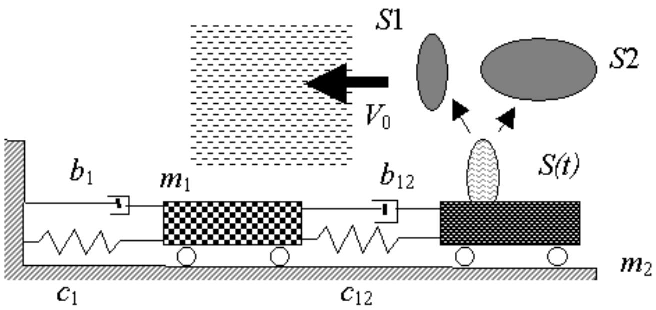

Fig. 1. **Scheme of model with area *S*(*t*) control:** m_1, m_2 – masses of moving bodies; c_1, c_{12} – stiffness of springs; b_1, b_{12} – damping coefficients; .V_0 – air flow velocity; S_1, S_2 – lower and upper level of area; of mass m_2; $S(t)$ – area control in time in phase plane (z_n – displacement, vz_n – velocity)

The criterion of optimization K is time T required to move object from initial position to end position. For system excitation any time must be solved the high-speed problem (2) [1 - 3]:

$$K = \int_{t_0}^{t_1} 1 \cdot dt \tag{2}$$

To assume $t_0 = 0$; $t_1 = T$, we have $K = T$. In this investigation equation of motion for large flow velocity $|V0| \ge |\dot{x}|$ may be described as (3):

$$\begin{aligned} m_1 \ddot{y} &= -c_1 y - c_{12}(y - z) - b_1 \dot{y} - b_{12}(\dot{y} - \dot{z}); \\ m_2 \ddot{z} &= c_{12}(y - z) + b_{12}(\dot{y} - \dot{z}) - u(t) \cdot (V_0 + \dot{z})^2, \end{aligned} \tag{3}$$

where $y, \dot{y}, \ddot{y}$ – displacement, velocity and acceleration of mass m_1;
$z, \dot{z}, \ddot{z}$ – displacement, velocity and acceleration of mass m_2.

To use new variables (phase coordinates) $x_1 = y$, $x_2 = \dot{x}_1 = \dot{y}$, $x_3 = z$, $x_4 = \dot{x}_3 = \dot{z}$ the system (6) may be written in first order differential equation form (4):

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= \frac{1}{m_1}[-c_1 x_1 - c_{12}(x_1 - x_3) - b_1 x_2 - b_{12}(x_2 - x_4)]; \\ \dot{x}_3 &= x_4; \\ \dot{x}_4 &= \frac{1}{m_2}[c_{12}(x_1 - x_3) + b_{12}(x_2 - x_4) - u(t)\cdot(V_0 + x_4)^2] \end{aligned} \tag{4}$$

In this system with two degree of freedom Hamiltonian is [1 -6]:

$$\begin{aligned} H = {}&\psi_0 + \psi_1 x_2 + \psi_2 (\frac{1}{m_1}\cdot\left(-c_1 x_1 - c_{12}(x_1 - x_3) - b_1 x_2 - b_{12}(x_2 - x_4)\right) + \psi_3 x_4 + \\ &+ \psi_4 (\frac{1}{m_2}(c_{12}(x_1 - x_3) + b_{12}(x_2 - x_4) - u(t)\cdot(V_0 + x_4)^2). \end{aligned} \tag{5}$$

here $H = \psi \cdot X$ where (4)

$$\psi = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{Bmatrix}; \; X = \begin{Bmatrix} 0 \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}. \tag{6}$$

Scalar multiplication of two last vector functions ψ and X in any time (function *H*) must be maximal. To have such maximum, control action *u(t)* must be within limits $u(t) = u_1$; $u(t) = u_2$, depending only from the sign of function ψ_4 (5):

$$\begin{aligned} &H = \max H, \\ &\text{if} \quad \psi_4 \cdot (-u(t)\cdot(V_0 + x_4)^2) = \max \end{aligned} \tag{7}$$

The main conclusion of optimal control law (8, 9) for this system with two degree of freedom is that value of area any time must be on the bounds (1): $S(t) = S_1$ or $S(t) = S_2$.

Synthesis of real control action

For realizing optimal control actions (in general case) system needs a feedback with two adapters: one for displacement measurement and another – for velocity measurement.

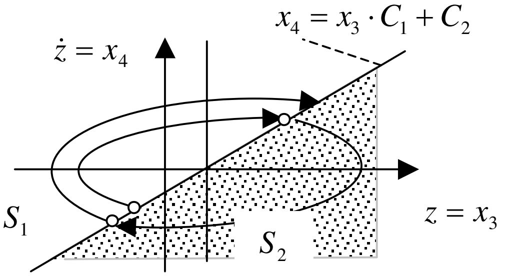

Fig. 2. **Control by two adapters and one separation line $x_4 = x_3 \cdot C_1 + C_2$ in phase plane:** x_3, x_4 – phase coordinates of mass m_2; C_1, C_2 – integration constants; S_1, S_2 – lower and upper level of area

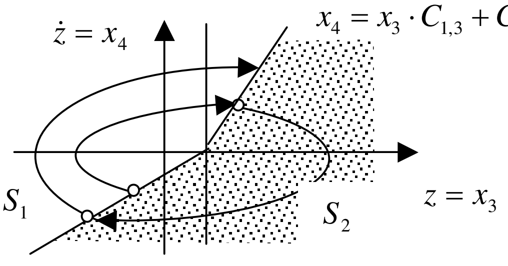

Fig. 3. **Control by two adapters and one broken separation line in phase plane:** *x*3, *x*4 – phase coordinates of mass m_2; $C_{1,3}$, C_2 – integration constants; S_1, S_2 - lower and upper level of area

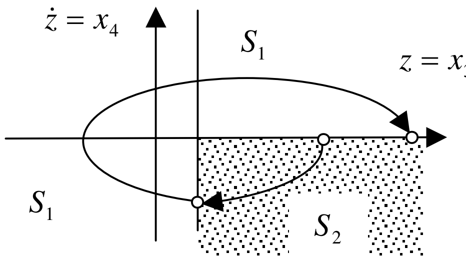

Fig. 4. **Control by two adapters with fixing only levels of phase coordinates:** x_3, x_4 – phase coordinates of mass m_2; S_1, S_2 - lower and upper level of area

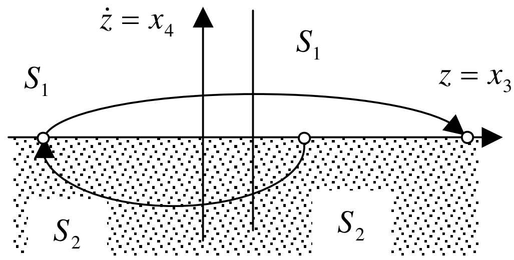

Fig. 5. **Control by only one velocity adapter which fix zero level:** x_3, x_4 – phase coordinates of mass m_2; S_1, S_2 - lower and upper level of area

Modeling equation for control action with fixing levels of both phase coordinates (Fig. 2.) is (8):

$$U = -\left[k \cdot (V0 + \dot{z})^2 \cdot S1 \cdot (F(z,\dot{z}))\right] - \left[k \cdot (V0 + \dot{z})^2 \cdot S2 \cdot (0{,}5 - 0{,}5 \cdot sign(\dot{z}))\right] \qquad (8)$$

where (see Equation 3)

$$U = -u(t) \cdot (V_0 + \dot{z})^2;$$

$$F(z,\dot{z}) = 1 - (0{,}5 - 0{,}5 \cdot sign(\dot{z}) \cdot \frac{1 + sign(z + C3)}{2});$$

k, C3 – constants.

Modeling equation for control action with one zero level of phase coordinate - velocity (Fig. 3.) is (9):

$$U = -\left[k \cdot (V0 + \dot{z})^2 \cdot S1 \cdot (0{,}5 + 0{,}5 \cdot sign(\dot{z}))\right] - \left[k \cdot (V0 + \dot{z})^2 \cdot S2 \cdot (0{,}5 - 0{,}5 \cdot sign(\dot{z}))\right] \qquad (9)$$

Motion modeling with constant excitation parameters

Results of modeling are shown in Fig. 6 - 13. Examples of motion for control (8) by two adapters with fixing only levels of phase coordinates are shown in Fig. 6 - 9. Motion is very stable because trajectories in phase plane for mass m2 practically do not crosse (Fig. 7.). Examples of motion in phase plane for second more efficiency control (9) (by only one velocity adapter which fix zero level) is shown in Fig. 10. - 13.

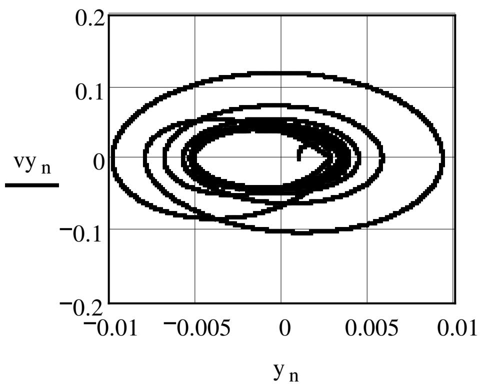

Fig. 6. **Motion of mass m_1 in phase plane with control (8) in the system SI**

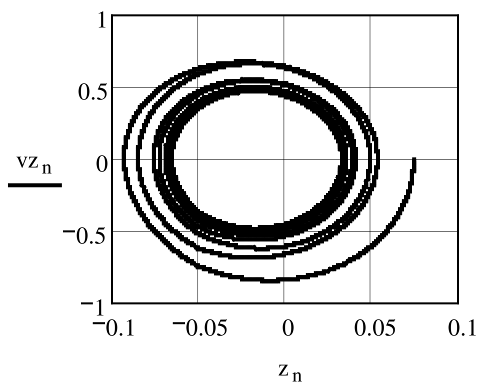

Fig. 7. **Motion of mass m_2 in phase plane with control (8) in the system SI**

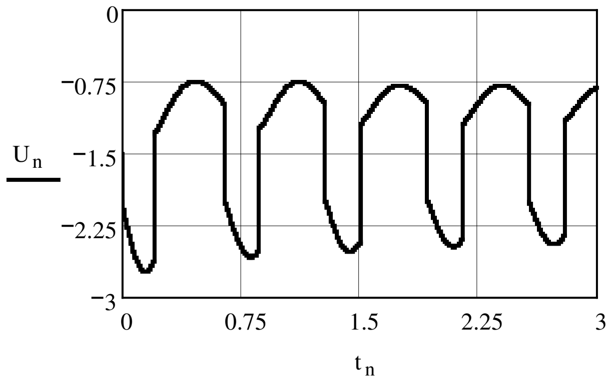

Fig. 8. **Control action (8) in time t_n domain in the SI system**

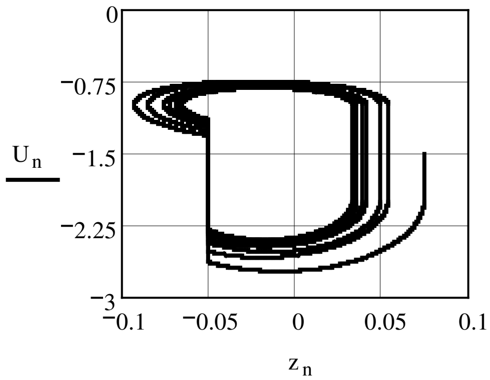

Fig. 9. **Control action (8) as function of displacement z_n in the SI system**

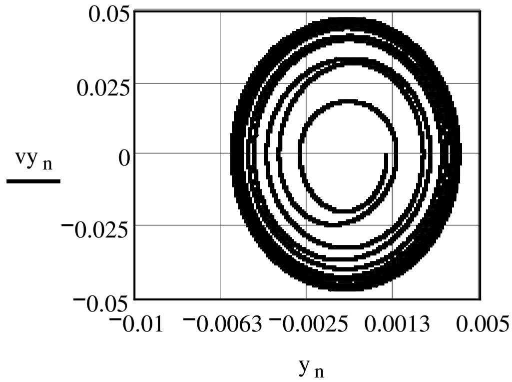

Fig. 10. **Motion of mass m_1 in phase plane (in SI system) with control (9). Motion is very stable**

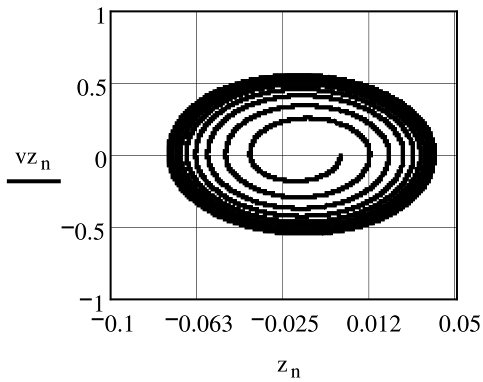

Fig. 11. **Motion of mass m_2 in phase plane (in SI system) with control (9). Motion is very stable**

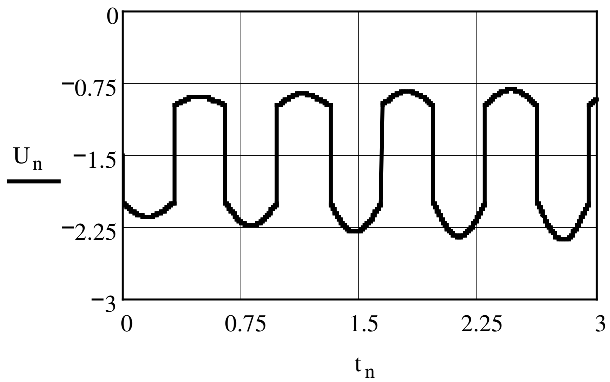

Fig. 12. **Control action (9) in time t_n domain (in SI system)**

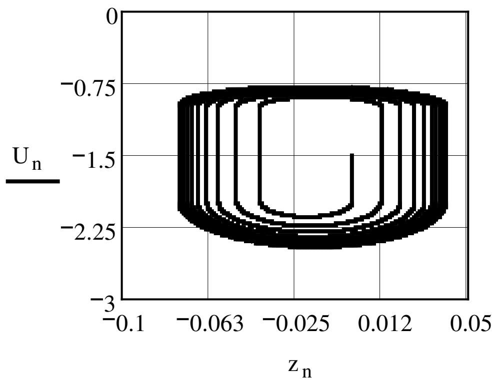

Fig. 13. **Control action (9) as function of displacement z (in SI system)**

Motion modeling with harmonica and random parameters excitation

To check up stability of motion mass m_2 was investigated two kinds of wind velocity exchange: by harmonica and random parameters. Investigation shows that motion is very stable (Fig. 14 -17).

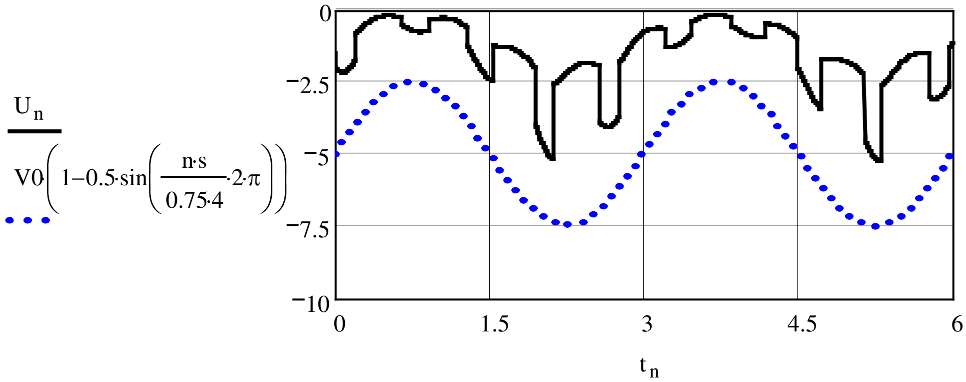

Fig. 14. **Control action (9) and wind velocity harmonica exchange in time t_n domain (in SI system)**

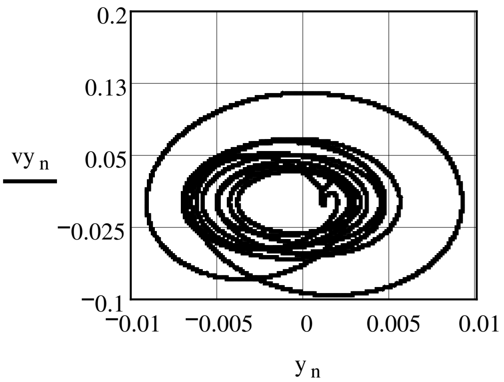

Fig. 15. **Motion of mass m $_1$ in phase plane (y_n – displacement, vy_n – velocity) with control Fig. 14 (in SI system)**

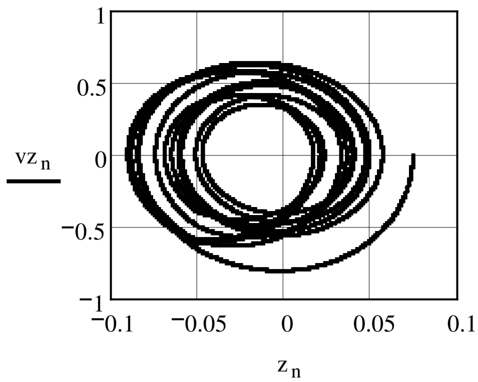

Fig. 16. **Motion of mass m $_2$ in phase plane (z_n – displacement, vz_n – velocity) with control Fig. 14 (in SI system)**

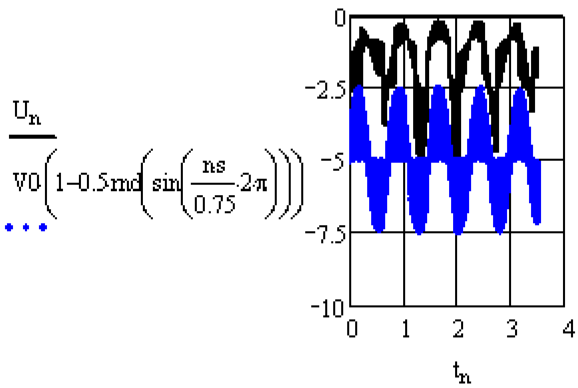

Fig. 17. **Control action (9) and wind velocity mixed exchange in time t_n domain**

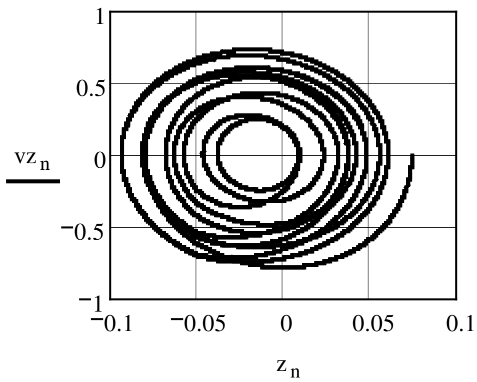

Fig. 18. **Motion of mass m_2 in phase plane (z_n – displacement, vz_n – velocity) with control (9)**

Analyses of controls (8) and (9) show that these adaptive systems have very stable motion (Fig. 10 - 18). It means that trajectories of main mass in phase plane practically dos not crossed and periodical cycle is achieved after small time.

At the end of investigations some experimental works inside wind tunnel are analyzed. Investigated system includes console spring and plane lamina at the end (Fig. 19). Experiments confirm that airflow excitation is very efficient and vibration was excited in large region of air flow velocity.

Fig. 19. **Wind tunnel and vibrating system inside (right) with console spring and plane lamina**

Conclusions

Air or water flow may be used for excitation objects motion in vibration technique. Control of object area allows finding very efficient mechatronic systems for energy conservation. Use of new vibration systems with air or water flow is in starting position and needs more fundamental investigations.

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