ADAPTIVE SELF-TUNING UP MODEL FOR NON-STATIONARY PROCESS SIMULATION

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Abstract. Methodology of non-linear and non-stationary process simulation, using MATLAB subprogram-SIMULINK, is expounded and justified. A self tuning up model to simulate the transient process of a non-linear and non-stationary electrical heater with variable electrical resistance, as a function of heater temperature, and changeable heat transfer coefficient, as a function of heater and air temperature, is viewed. To compare the transient process of a simplified stationary model with constant sensitivity and inertia factors, and the real transient process of a non-stationary model with time dependent sensitivity and inertia factors, the simulation model, applying Laplace transforms and SIMULINK library components, are composed. The transient process simulation is made for the step case input voltage, for the linear changing input voltage and for the step and random changing input voltage. The analyses of the simulation results show that the real transient process of the electrical heater non-stationary model differs from the transient process of the simplified stationary model substantially.

Keywords: non-stationary object, self-tuning model, virtual analyses, sensitivity factor, inertia factor, simulation.

Introduction

Several technological objects, such as waste water aeration tanks, electrical machine systems and cogeneration plants (internal combustion engine and electrical generator), are typical non-linear and non-stationary objects with time dependent transfer coefficients (gains) and time constants [1-3]. In that case the transient characteristics of the object are described by non-linear and non-stationary differential equations, mathematical analyses of which are problematic. Simplification of mathematical models, applying linearization and "freezing" of variable coefficients, put into practice, causes an incorrect result.

Virtual analyses, applying SIMULINK [4-6], makes it possible to compile the simulation models of such objects without simplifications and therefore allow to obtain the transient characteristics with substantially higher accuracy because of automatic variations of the transfer coefficients and of the time constants during simulation time. For this purpose the on-line continuous adaptive feedbacks or the off-line discrete feedbacks with automatic switchboard [3] may be employed.

Main methodological tasks of this work are to develop and to investigate an adaptive self-tuning up simulation model of the electrical heating object using dynamic on-line feedbacks for sensitivity and inertia factors adoption to the variable transient temperature during the whole simulation process.

Materials and methods

1. Mathematical and simulation models of electrical heating object

The research object is a two volume electrical thermostat, which consists of an electrical heater (300 W) and thermally isolated air mass. Two different models of electrical thermostats are compared: 1) the stationary model with constant transfer coefficients and invariable time constants both of heater and air mass; 2) the non-stationary model with temperature dependent transfer coefficient and time constant of heater and constant transfer coefficient and time constant of air mass The mathematical models, simulation block-diagrams and characteristics of the research object are given in the text.

The operator equations and the transfer functions of the heater and the air mass are composed using mathematical analyses, operator mathematics and Laplace transforms [4]. The transient process simulation in Windows environment is performed using SIMULINK [5, 6]. Variable sensitivity and inertia factors of the electrical heater are calculated according to their technical parameters, applying analytical and empirical expressions [4, 7, 8].

For simplicity the transient heat transfer process in the electrical heating object is analysed as a lumped process [7], where temperature of the medium changes uniformly with time, not position.

Then the transient heat transfer of the electrical heater and air mass can be described by the common differential equations. Using Laplace transforms to differential equations, we obtain the operator equations and the transfer functions for transient temperature simulation as a function of input voltage.

The operator equation and the transfer function of the stationary model for transient process simulation in the electrical heater:

$$T_h^* \times \tau_h(s)^* \times s + \tau_h(s)^* = K_h^* \times U^2(s),\ W_h^*(s) = \frac{\tau(s)^*}{U^2(s)} = \frac{K_h^*}{T_h^* \times s + 1}, \quad (1)$$

where $T_h^* = 8.1$ min – average time constant of the heater;
$\tau_h(s)^*$ – Laplace transform from surface over-temperature ($\tau_h^* = \Theta_h^* - \Theta_0$);
Θ_h^*, Θ_0 – variable surface average temperature and initial temperature of the heater and air mass, °C;
$K_h^* = 0.007$ °C·V^{-2} – average transfer coefficient of the heater;
$U^2(s)$ – Laplace transform from input step case squared voltage U^2, V^2;
s – Laplace variable, min^{-1}.

The operator equation and the transfer function of the stationary model for transient process simulation in air mass:

$$T_a^* \times \tau_a(s)^* \times s + \tau_a(s)^* = K_a^* \times \tau_h(s)^*,\ W_a^*(s) = \frac{\tau_a(s)^*}{\tau_h(s)^*} = \frac{K_a^*}{T_a^* \times s + 1}, \quad (2)$$

where $T_a^* = 5.5$ min – average time constant of the air mass;
$\tau_a(s)^*$ – Laplace transform from air mass over-temperature ($\tau_a^* = \Theta_a^* - \Theta_0$);
Θ_a^* – variable air mass average temperature, °C;
$K_a^* = 0.4$ – average transfer coefficient of the air mass.

Actually, the electrical heater is a non-stationary object because of the changeable heating time constant $T_h = f(\Theta_h, \Theta_a)$ as well as due to the fluent transfer coefficient $K_h = f(\Theta_h, \Theta_a)$, which changes during the whole transient heat transfer process. The expression for T_h calculation using the parameters of the actual electrical heater is as follows [4]:

$$T_h = \frac{c \cdot m}{h(\Theta_h, \Theta_a) \cdot A}. \quad (3)$$

where $c = 950$ J· (kg·°C)$^{-1}$ – specific heat;
$m = 0.45$ kg – mass;
$A = 0.052$ m^2 – surface area;
$h(\Theta_h, \Theta_a)$ – surface both of convection and radiation heat transfer coefficient, W·(m^2·°C)$^{-1}$.

The surface heat transfer coefficient can be calculated by empirical formula [8]:

$$h \approx 9.3 + 0.047 \cdot (\Theta_h - \Theta_a) + 0.021 \cdot \sqrt{\Theta_h - \Theta_a}, \quad (4)$$

where $h_o = 9.3$ W· (m^2·°C)$^{-1}$ – initial heat transfer coefficient both of convection and radiation.

Since T_h is a function of time dependent variables Θ_h and Θ_a, T_h is named as a process inertia factor.

The expression for K_h calculation using the parameters of the actual electrical heater is as follows [4]:

$$K_h = \frac{1}{h(\Theta_h, \Theta_a) \cdot A \cdot R(\Theta_h)}. \quad (5)$$

where $R(\Theta_h)$ – electrical resistance of electrical heater, Ω.

The nichrome heater resistance can be calculated by analytical formula:

$$R(\Theta_h) = R_{\Theta_0} \frac{1 + \alpha_n \cdot \Theta_h}{1 + \alpha_n \cdot \Theta_0}. \tag{6}$$

where $\alpha_n = 0.0004$ °C^{-1} – resistance-temperature coefficient of nichrome heater;
$R_{\Theta 0} = 161\ \Omega$ – initial resistance of nichrome heater, if $\Theta_0 = 20$ °C.

Since K_h is a function of time dependent variables Θ_h and Θ_a, is named K_h as a process sensitivity factor.

The non-stationary and non-linear mathematical model for output temperature $\Theta_a(t)$ simulation in the electrical dryer, assuming that the heated air mass is a stationary object with constant parameters K_a*, T_a*, is as follows:

$$\Theta_a(s) = \frac{K_h(\Theta_h, \Theta_a)}{T_h(\Theta_h, \Theta_a) \times s + 1} \times \frac{K_a*}{T_a * \times s + 1} \times U^2(s) + \Theta_0. \tag{7}$$

where $\Theta_a(s)$ – Laplace transform of air mass transient temperature.

It is unable to solve the given equation (7) analytically because of non-stationary coefficients and non-linear input variable. Further it will be shown how to solve a problem virtually, using SIMULINK.

The block diagram of the non-stationary and non-linear model for simulation of transient temperatures $\Theta_h(t)$ and $\Theta_a(t)$ of the electrical heater, described by equation (7), is shown in Figure 1. The adaptive self-tuning up model of the electrical heater consists of the "Adaptive R-module" and "Adaptive h-module" for dynamic calculation of the heat transfer coefficient h and electrical resistance R during the transient process of temperatures $\Theta_h(t)$, $\Theta_a(t)$, according to expressions (4, 6).

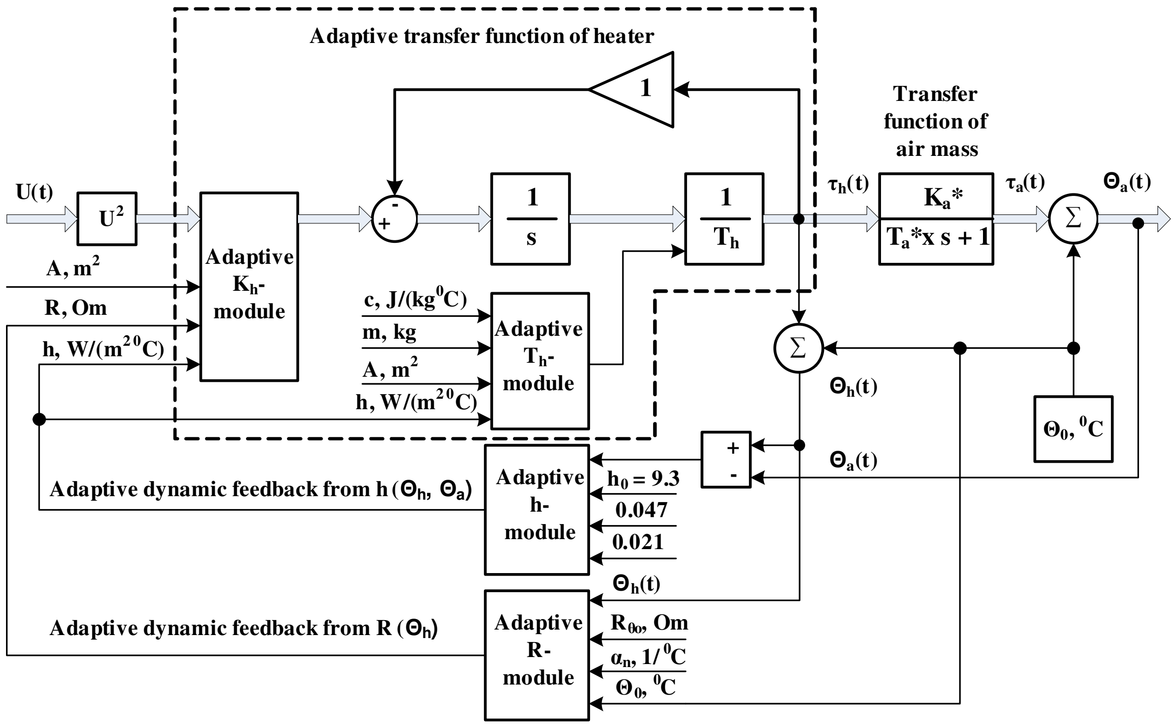

Fig. 1. **Adaptive self - tuning up model of electrical heater with adaptive dynamic feedbacks from temperature dependent heat transfer coefficient *h* (Θ_h, Θ_a) and from heater resistance *R* (Θ_h)**

The "Adaptive R-module" creates dynamic feedback $R(\Theta_h)$ for the sensitivity factor K_h calculation. The "Adaptive h-module" forms dynamic feedback for both K_h and the inertia factor T_h

calculation. The adaptive dynamic feedbacks allow calculation of K_h and T_h step by step during the simulation time.

The adaptive self-tuning up transfer function of the electrical heater is composed of the following modules: “Adaptive K_h-module”; “Adaptive T_h-module”; “1/*s*” – integrator; “1/T_h” – module for T_h reversion; “1” – unit negative feedback.

2. Block diagram for transient process simulation of stationary and non -stationary models

The simulation block-diagram of the electrical thermostat is compiled using standard blocks from SIMULINK libraries (Figure 2). The block-diagram consists of several modules for automatic calculations and simulation: – the module for calculation of the heat transfer coefficient h as a function of the heater and air mass temperatures $h = f(\Theta_h, \Theta_a)$ (4); – the module for calculation of the heater resistance as a function of the heater temperature $R = f(\Theta_h)$ (6) T_h – the module for calculation of the inertia factor T_h dependent on fixed parameters and the fluent heat transfer coefficient h (Θ_h, Θ_a) (3); K_h x U^2 – the module for calculation of the sensitivity factor K_h dependent on fixed parameters and fluent parameters $h(\Theta_h, \Theta_a)$, $R(\Theta_h)$ (5), and for K_h multiplication by input variable – squared voltage U^2 ; Thermostat – module for output temperature $\Theta_a = f(t)$ simulation accordingly to equation (7).

The transient process simulation is made for the step case input voltage, for the linear growing input voltage and for the step and random input voltage. For appropriate input voltage formation the following functional blocks are used: “Switch 1” and “Switch 2” – manual switches; “U_{step}” – step case voltage generator; “U_{linear}” – linear growing voltage generator; “ΔU_{random}”– random normally distributed voltage generator; “Saturation” – for linear growing voltage limitation.

Digital displays for input and output volumes visualization: U_s – input steady voltage, V; Θ_{as}^*, Θ_{as} – air mass steady temperature for stationary and non-stationary models, ˚C; Θ_{hs}^*, Θ_{hs} – heater steady temperature for stationary and non-stationary models, ˚C. “Scope 1” and “Scope 2” for visualization of input and output transient characteristics – $U = f(t)$; $\Theta_a^* = f(t)$; $\Theta_a = f(t)$; $\Theta_h^* = f(t)$; $\Theta_h = f(t)$.

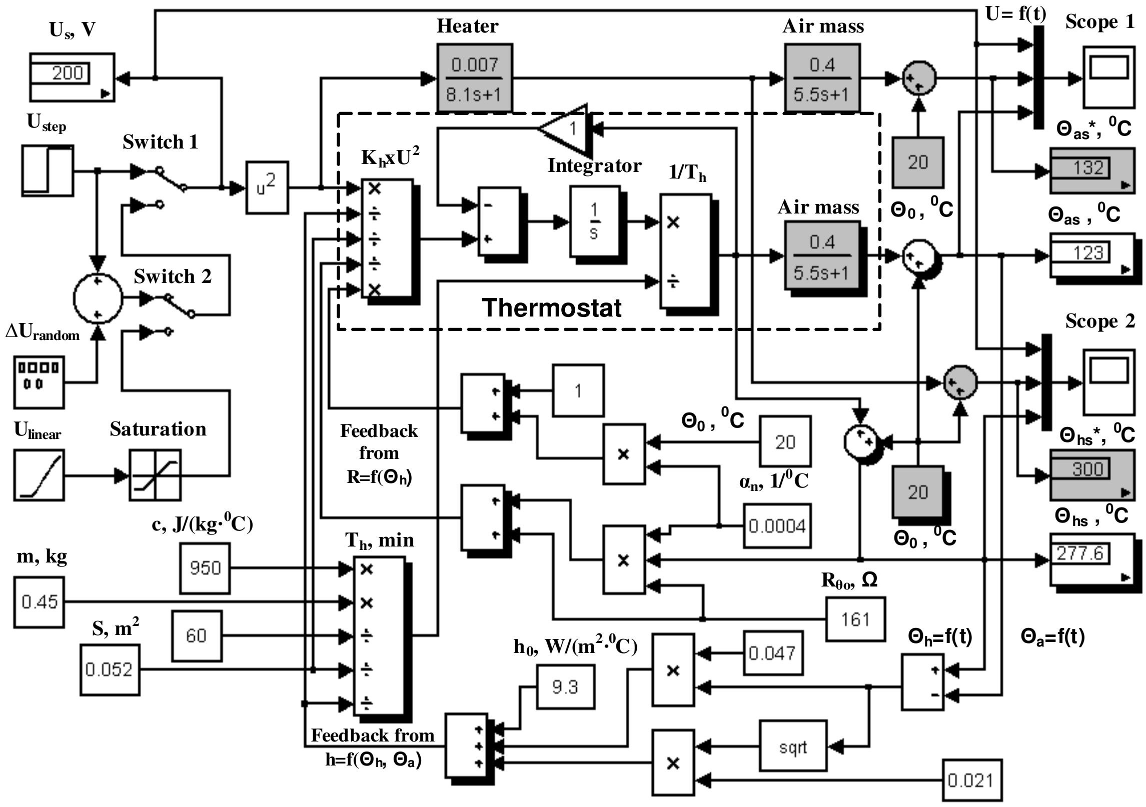

Fig. 2. **Block diagram for transient process simulation of stationary and non-stationary electrical heater models with calculation of variable sensitivity and inertia factors during simulation**

Discussion and results

The simulated characteristics of stationary and non-stationary models for step and linear growing input voltage are shown in Figures 3-5. To form the step case voltage U_1 = 100 V and linear growing voltage from U_1 = 100 V to limited value U_2 = 200 V, the "U_{linear}" and "Saturation" blocks have been activated. For this purpose "Switch 1" and "Switch 2" should be set in the lower position (Figure 2). The configuration parameters are as follows: for the "U_{linear}" block (initial voltage – 100 V, initial delay time – 40 min, slope – 50 V·min^{-1}); for the "Saturation" block (minimum voltage – 0 V, maximum voltage – 200 V).

The simulation shows that the sensitivity factor K_h and the inertia factor T_h of the heater are not constant, but change from the maximum value $K_{h\ \max}$ = 12.5·10^{-3} °C·V^{-2}, $T_{h\ \max}$ = 14.6 min at initial temperature 20 °C to the minimum one $K_{h\ \min}$ = 6.4·10^{-3} °C·V^{-2}, $T_{h\ \min}$ = 8.15 min at maximum temperature 278 °C during the whole transient process of heating (Figure 3). Therefore, only the non-stationary model is able to ensure correct heating results.

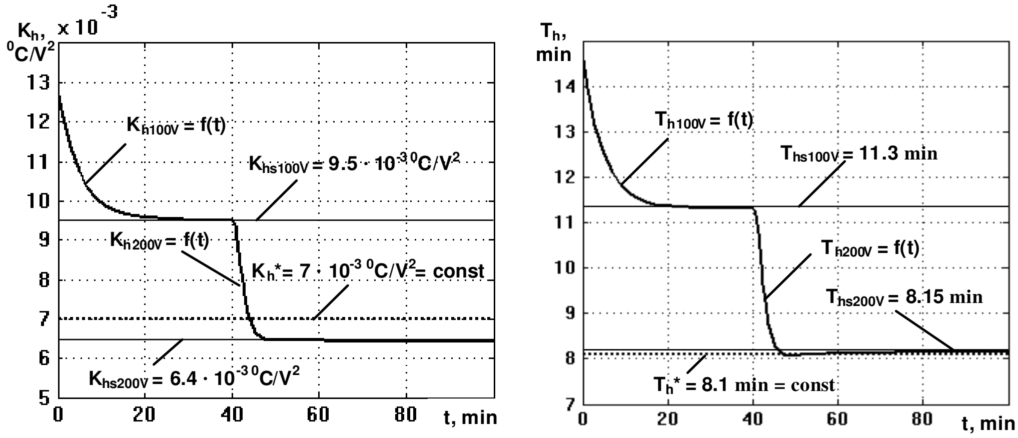

Fig. 3. **Characteristics of sensitivity and inertia factors for stationary model (K_h^* = 7·10^{-3} °C·V^{-2}= =const and T_h^* = 8.1 min = const) and for non-stationary self-tuning up model ($K_{h\ 100\ V} = f(t)$, $K_{h\ 200\ V} = f(t)$ and $T_{h\ 100\ V} = f(t)$, $T_{h\ 200\ V} = f(t)$) under two step voltage (U_1 = 100 V, U_2 = 200 V)**

The simulated response of the heater temperature for the step case initial voltage, which is rapidly growing from the first step – 100 V to the limited second step – 200 V after delay time – 40 min (necessary for temperature stabilization at the first step voltage) demonstrates, that the transient characteristic $\Theta_h^* = f(t)$, if the simplified stationary model is applied, substantially differs from that $\Theta_h = f(t)$, which is obtained using the adaptive self-tuning up non-stationary model (Figure 4). The real heating inertia factor T_h and the sensitivity factor K_h for the non-stationary model are higher at low temperatures because of a lower heat transfer coefficient h. Therefore, the heating proceeds slower, but steady temperature is higher by the side of that for the stationary model. Everything is opposite at high temperatures.

The simulated characteristics of stationary and non-stationary models for step and random input voltage – U_{step} and ΔU_{random} are shown in Figures 6-8. To get the step case voltage U_{step} and normally distributed random voltage component ΔU_{random}, the "U_{step}" and "ΔU_{random}" blocks with "Adder" should be activated. For this purpose "Switch 1" should be set in the lower position, but "Switch 2" – in the over position (Figure 2). The configuration parameters are as follows: for the "U_{step}" generator (initial value – 0 V, final value – 200 V, step time – 0 min); for the "ΔU_{random}" generator (amplitude – ±20 V, frequency – 0.25 min^{-1}).

The simulation shows that the sensitivity factor K_h and inertia factor T_h of the heater fluctuate randomly around the balance value $K_{h\ balance}$ = 6.35 · 10^{-3} °C·V^{-2}, $T_{h\ balance}$ = 8.03 min on ±5 % with the same frequency as temperature, but counter phase to temperature oscillations (Figure 6). Only an adaptive self-tuning up model is able to simulate randomly fluctuating temperature of the heater and air mass correctly.

The simulated response of the heater temperature for the randomly fluctuating input voltage 200 V ± 10 % with amplitude ±20 V and frequency 0.25 min^{-1} testifies, that the output temperature of the adaptive non-stationary model $\Theta_h = f(t)$ is more sensitive to voltage fluctuations as that $\Theta_h^* = f(t)$ of the non-adaptive stationary model.

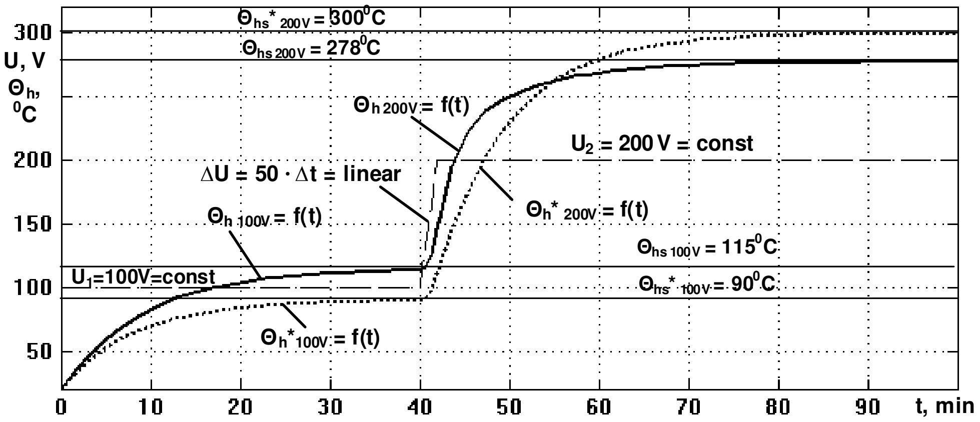

Fig. 4. **Simulated characteristics of heater temperature for stationary model ($\Theta_h^* = f(t)$) and for non-stationary self-tuning up model ($\Theta_h = f(t)$) under step and linear growing voltage ($U_1 = 100$ V, $\Delta U = 50 \cdot \Delta t$, $U_2 = 200$ V)**

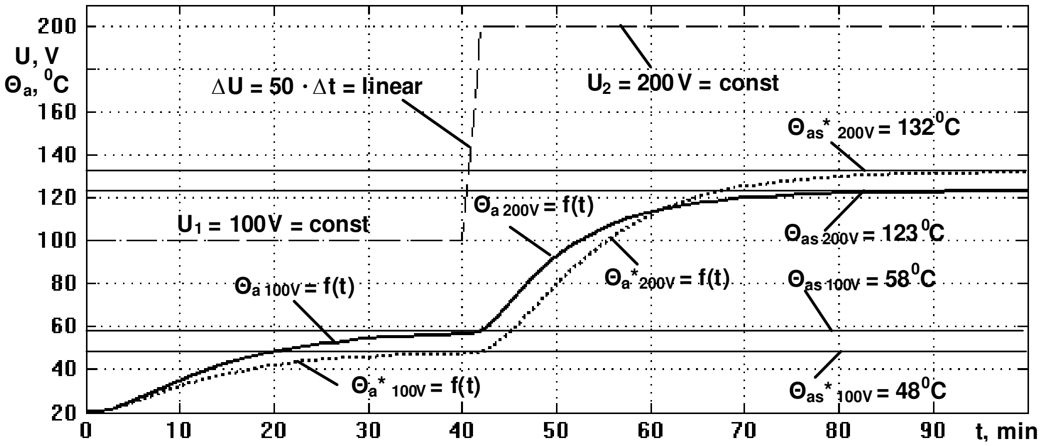

Fig. 5. **Simulated characteristics of air mass temperature for stationary model ($\Theta_a^* = f(t)$) and for non-stationary self-tuning up model ($\Theta_a = f(t)$) under step and linear growing voltage ($U_1 = 100$ V, $\Delta U = 50 \cdot \Delta t$, $U_2 = 200$ V)**

Due to the heating inertia, the random fluctuations of the voltage up to ±10 % cause less temperature deviations of the heater – ±6 % (Figure 7) and still less of the air mass – ±2.5 % (Figure 8). Therefore, the heating inertia of the heater and of air mass operates as a filter of input fluctuations.

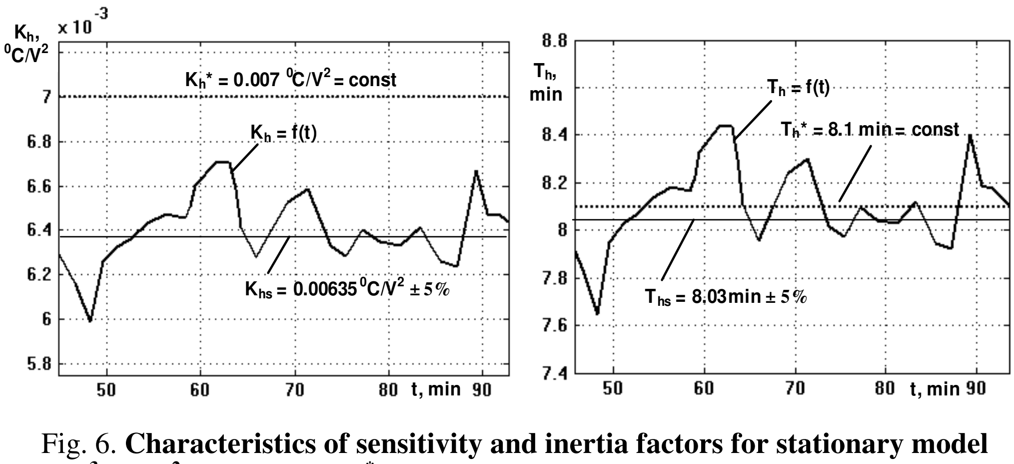

Fig. 6. **Characteristics of sensitivity and inertia factors for stationary model ($K_h^* = 7 \cdot 10^{-3}$ °C·V^{-2} = const and $T_h^* = 8.1$ min = const) and for non-stationary self-tuning up model ($K_h = f(t)$, and $T_h = f(t)$) under random variable voltage ($U = 200$ V ± 10 %)**

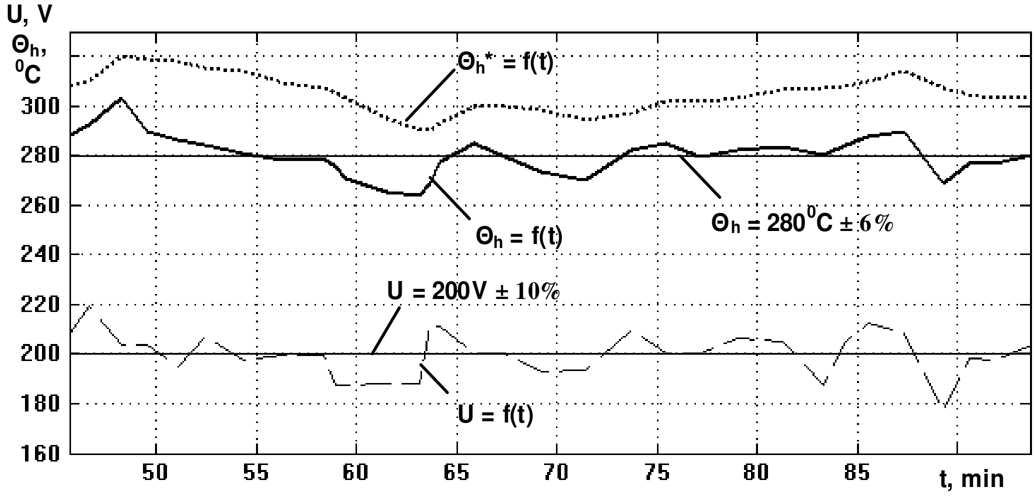

Fig. 7. **Simulated characteristics of heater temperature for stationary model ($\Theta_h^* = f(t)$) and for non-stationary self-tuning up model ($\Theta_h = f(t)$) under random variable voltage (U = 200 V±10 %)**

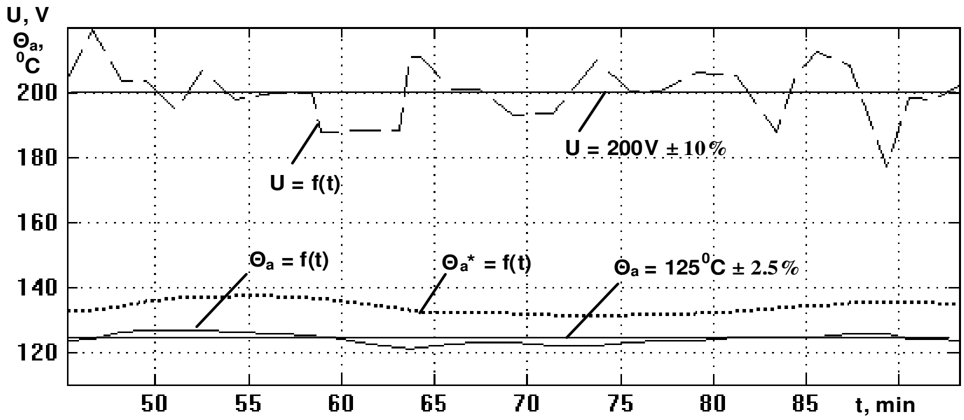

Fig. 8. **Simulated characteristics of air mass temperature for stationary model ($\Theta_a^* = f(t)$) and for non-stationary self-tuning up model ($\Theta_a = f(t)$) under random variable voltage (U = 200 V ± 10 %)**

Conclusions

1. For transient process virtual analyses with appropriate accuracy in non-linear and non-stationary technological objects, the sensitivity and inertia factors of which change substantially on output variables, the adaptive self-tuning up simulation model should be developed applying dynamic feedbacks from output variables for automatic adoption of the functionally dependent components of the model during the whole simulation process.
2. The simulation results of the given electrical heater using the adaptive self-tuning up model show, if the input voltage changes from U_1 = 100 V to U_2 = 200 V, the steady sensitivity factor of the heater decreases from $K_{hs\ 100\ V} = 9.5 \cdot 10^{-3}$ °C·V^{-2} to $K_{hs\ 200\ V} = 6.4 \cdot 10^{-3}$ °C·V^{-2} and the steady inertia factor decreases from $T_{hs\ 100\ V}$ = 11.3 min to $T_{hs\ 200\ V}$ = 8.15 min, what causes a relative difference of the simulated steady final temperature Θ_{hs} up to 25 % in comparison with the Θ_{hs}^*, what is obtained using the simplified stationary model.

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