APPROXIMATE CALCULATION ALGORITHM FOR SINGLE PHASE BRIDGE RECTIFIER WITH CAPACITOR FILTER

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Abstract. In the paper approximate calculation algorithm for single phase rectifier with C filter is described. The offered algorithm contains formulas which allow determining capacitor current, total power factor as well as capacitor rated ripple current and equivalent series resistance without referring to datasheet. Although the formulas used in this algorithm are based on simplified mathematical models, there are influential circuit parameters observed such as transformer leakage inductive reactance, diode slope resistance and diode commutation interval. The algorithm corresponds to the analyzer style. It means that there are completely given single phase transformer, diodes, capacitor battery parameters and load resistance, and the analyzer finds out voltages, currents and powers. The algorithm contains only evaluated formulas so there is no need for advanced mathematics operations. It is tested on MS Excel. Calculation in Excel is not fully automated: there are two manually operable cells for leading the iteration process to its convergence.

Keywords: capacitor filter, rectifier, power factor, ripple current, calculation algorithm.

Introduction

The authors of the paper had a target to discover a full and easy-to-use analyzing algorithm for a single phase bridge rectifier with a capacitor filter. As readily available calculation tool software MS Excel was chosen. So, the formulas in the algorithm had to answer to Excel functionality. It means that the formulas must not contain advanced mathematical operations. As a base for the new development a simple algorithm from the reference was taken [1].

Reference analysis

One of the key formulas which were taken from the reference [1] is formula (12) in its modified form – peak value of secondary winding current. The second formula in its modified form is (15) – conduction interval of secondary current. The third formula in its modified form is (16) – average output voltage. Formulas (11), (13), (36), (38) are taken from the reference [2]. Formula (31) is common in several capacitor manufacturer datasheets.

Structure of the algorithm

The introduced formulas are (5), (19), (20), (24), (27), (28), (33), (48). The rest of the formulas are basic electrical engineering formulas. In the formula evaluating process online calculation tool Wolphram Alpha was used. As a circuit simulator 5SPICE was used.

Due to the algorithm complexity there is a necessity for the iterative process. Since software EXCEL cannot automatically calculate such a problem without specially programmed macros, there must be break points introduced to avoid "Circular Reference" error. In EXCEL spreadsheet values from formulas (13) and (17) are manually operated to get the iteration process to its convergence. Consequently the algorithm is semi-automatic, however, it can be supplemented with specially programmed macros which allow obtaining fully automated operation.

The input values are presented as the following physics exercise. The given single phase transformer connected to $f = 50$ Hz AC grid. No load voltage at the secondary winding is equal to $V_{20} = 25$V, short circuit resistance $R_T = 0.26$ Ω, short circuit reactance $X_T = 0.23$ Ω. Also the given single phase bridge rectifier with one diode threshold voltage $V_{(T0)} = 0{,}78$V and slope resistance $r_T = 0.055$ Ω. At the rectifier output a capacitor battery is connected which contains $N_{CP} = 1$ capacitors in parallel and $N_{CS} = 1$ capacitors in series. Each capacitor capacitance $C = 16500$ μF and rated voltage $V_{CR} = 50$ V. Load resistance $R_{load} = 5.91$ Ω. All formulas operate correctly with SI units.

Formula series which form the mentioned algorithm follow here.

AC grid oscillation period T:

$$T = \frac{1}{f}. \tag{1}$$

Total resistance of transformer and diode R_Σ:

$$R_\Sigma = R_T + r_T. \tag{2}$$

Total impedance of transformer and diode Z_Σ:

$$Z_\Sigma = \sqrt{R_\Sigma^2 + X_T^2}. \tag{3}$$

Equivalent capacitance of capacitor battery C_{ekv}:

$$C_{ekv} = C \cdot \frac{N_{CP}}{N_{CS}}. \tag{4}$$

Equivalent series resistance of single capacitor ESR:

$$ESR = \frac{0.02}{C \cdot V_{CR}}. \tag{5}$$

Equivalent series resistance of capacitor battery ESR_{ekv}:

$$ESR_{ekv} = ESR \cdot \frac{N_{CS}}{N_{CP}}. \tag{6}$$

Capacitive reactance of capacitor battery X_C:

$$X_C = \frac{1}{2\pi \cdot f \cdot C}. \tag{7}$$

Total resistance in circuit $R_{\Sigma C}$:

$$R_{\Sigma C} = R_\Sigma + ESR_{ekv}. \tag{8}$$

Total impedance in circuit $Z_{\Sigma C}$:

$$Z_{\Sigma C} = \sqrt{R_{\Sigma C}^2 + X_T^2}. \tag{9}$$

No load peak voltage of secondary winding V_{20p}:

$$V_{2PK} = \sqrt{2} \cdot V_{20}. \tag{10}$$

Commutation interval T_{com}:

$$T_{com} = \frac{T}{2\pi} \cdot \arccos\left(1 - \frac{I_d \cdot Z_\Sigma}{U_{20P}}\right), \tag{11}$$

where I_d – load current.

Peak value of secondary winding current I_{2PK}:

$$I_{2PK} = \frac{V_{d0}}{\sqrt{2 \cdot \sqrt{R_{\Sigma C}^2 + (X_T - X_C)^2} \cdot R_{load}}}. \tag{12}$$

Peak forward voltage on single rectifier diode V_F:

$$V_F = V_{(T0)} + I_{2PK} \cdot R_d. \tag{13}$$

No load voltage on capacitor battery V_{d0}:

$$V_{d0} = V_{2PK} - 2 \cdot V_F. \tag{14}$$

Conduction interval of secondary current T_{CI}:

$$T_{CI} = \frac{T}{2} \cdot \sqrt[4]{\frac{Z_{\Sigma C}}{2 \cdot R_{load}}} + \frac{T_{com}}{2} . \tag{15}$$

Average output voltage at load V_{dAVG}:

$$V_{dAVG} = V_{d0} \cdot \left(1 - \sqrt{\frac{Z_{\Sigma}}{2 \cdot R_{load}}}\right) . \tag{16}$$

Load current I_d:

$$I_d = \frac{V_{dAVG}}{R_{load}} . \tag{17}$$

Conduction angle of secondary current θ_{CI}:

$$\theta_{CI} = \frac{T_{CI}}{T} \cdot 2\pi . \tag{18}$$

Short circuit peak current $I_{2SPK\,()}$:

$$I_{2SPK} = \frac{I_{2PK}}{1 - \sin\left(\frac{\pi - \theta_{CI}}{2}\right)} . \tag{19}$$

Output voltage increasing angle θ_{CU}:

$$\theta_{CU} = \pi - 2 \cdot \arcsin\left[\sin\left(\frac{\pi - \theta_{CI}}{2}\right) + \frac{I_d}{I_{2SPK}}\right] . \tag{20}$$

Output voltage increasing interval T_{CU}:

$$T_{CU} = \theta_{CU} \frac{T}{2\pi} . \tag{21}$$

Output voltage decreasing interval T_{DU}:

$$T_{DU} = \frac{T}{2} - T_{CU} . \tag{22}$$

Output voltage ripple swing ΔV_d:

$$\Delta V_d = \frac{I_d T_{DU}}{C_{ekv}} . \tag{23}$$

Output voltage minimum V_{dmin}:

$$V_{dmin} = V_{dAVG} - \Delta V_d \cdot \left(\frac{3}{4} - \frac{T_{CU}}{T}\right) . \tag{24}$$

Maximum output voltage V_{dmax}:

$$V_{dmax} = V_{dmin} + \Delta V_d . \tag{25}$$

Output voltage relative ripple $\Delta V_{d\%}$:

$$\Delta V_{d\%} = \frac{\Delta V_d}{V_d} \cdot 100\% . \tag{26}$$

Secondary RMS current I_{2RMS}:

$$I_{2\text{RMS}} = \sqrt{\frac{1}{2\pi}\left[\theta_{\text{CI}}\left(3I_{2\text{SPK}}^2 + 4I_{2\text{SPK}}I_{2\text{PK}} + 2I_{2\text{PK}}^2\right) + I_{2\text{SPK}}^2 \sin\theta_{\text{CI}} + 8I_{2\text{SPK}}\left(I_{2\text{PK}} - I_{2\text{SPK}}\right)\sin\frac{\theta_{\text{CI}}}{2}\right]}. \quad (27)$$

Secondary average current $I_{2\text{AVG}}$:

$$I_{2\text{AVG}} = \frac{1}{\pi}\left[\theta_{\text{CI}}\left(I_{2\text{PK}} - I_{2\text{SPK}}\right) + 2I_{2\text{SPK}}\sin\frac{\theta_{\text{CI}}}{2}\right]. \quad (28)$$

Secondary RMS current in diode I_{VDRMS}:

$$I_{\text{VDRMS}} = \frac{I_{2\text{RMS}}}{\sqrt{2}}. \quad (29)$$

Secondary average current in diode I_{VDAVG}:

$$I_{\text{VDAVG}} = \frac{I_{2\text{AVG}}}{2}. \quad (30)$$

Single capacitor leakage current I_{LEAK}:

$$I_{\text{LEAK}} = 0.03 \cdot C \cdot V_{\text{CR}}. \quad (31)$$

Capacitor battery leakage current $I_{\text{LEAK}\Sigma}$:

$$I_{\text{LEAK}\Sigma} = N_{\text{CP}} \cdot I_{\text{LEAK}}. \quad (32)$$

Capacitor rated ripple current I_{RIPPLE}:

$$I_{\text{RIPPLE}} = 20.3 \cdot C^{0.79} \cdot V_{\text{CN}}^{0.5}. \quad (33)$$

Capacitor battery RMS current $I_{\text{CRMS}\Sigma}$:

$$I_{\text{CRMS}\Sigma} = \sqrt{I_{2\text{RMS}}^2 - I_{\text{d}}^2}. \quad (34)$$

Power losses in transformer ΔP_{TR}:

$$\Delta P_{\text{TR}} = I_{2\text{RMS}}^2 \cdot R_T. \quad (35)$$

Power losses in single diode ΔP_{VD1}:

$$\Delta P_{\text{VD1}} = V_{(\text{T0})} \cdot I_{\text{VDAVG}} + I_{\text{VDRMS}}^2 \cdot r_{\text{T}}. \quad (36)$$

Total power losses in four diodes ΔP_{VD4}:

$$\Delta P_{\text{VD4}} = 4 \cdot \Delta P_{\text{VD1}}. \quad (37)$$

Power losses in capacitor ΔP_{C}:

$$\Delta P_{\text{C}} = V_{\text{dAVG}} \cdot I_{\text{LEAK}\Sigma} + I_{\text{CRMS}\Sigma}^2 \cdot ESR_{\text{EKV}}. \quad (38)$$

Secondary winding RMS voltage at given load:

$$V_{2\text{RMS}} = V_{20} - I_{2\text{RMS}} \cdot R_{\text{T}}. \quad (39)$$

RMS current in single capacitor:

$$I_{\text{CRMS}} = \frac{I_{\text{CRMS}\Sigma}}{N_{\text{CP}}}. \quad (40)$$

Maximum voltage on single capacitor:

$$V_{\text{Cmax}} = \frac{V_{\text{dmax}}}{N_{\text{CS}}}. \quad (41)$$

Load power:

$$P_{d} = V_{dAVG} \cdot I_{d}.$$ (42)

Total consumed active power:

$$P_{1} = P_{d} + \Delta P_{TR} + \Delta P_{VD4} + \Delta P_{C}.$$ (43)

Total from grid consumed apparent power:

$$S_{1} = V_{20} \cdot I_{2RMS}.$$ (45)

Total from transformer consumed apparent power:

$$S_{2} = V_{2RMS} \cdot I_{2RMS}.$$ (46)

Total power factor:

$$\chi_{1} = \frac{P_{1}}{S_{1}}.$$ (47)

Displacement power factor:

$$\cos\varphi_{1} = \cos\left(\frac{T_{com}}{T} \cdot 2\pi\right).$$ (48)

Distortion power factor:

$$\nu_{1} = \frac{\chi_{1}}{\cos\varphi_{1}}.$$ (49)

Efficiency:

$$\eta = \frac{P_{d}}{P_{1}}.$$ (50)

Calculation result comparison

5SPICE was used to compare some of the calculation results. SPICE circuit component parameters are visible in Figure 1, while the corresponding simulation graph is visible in Figure 2. The red line is I_{2} curve, orange line is V_{d} curve while the green line is V_{2} curve. In the circuit diode bridge GPBC1510 was used. Approximate SPICE parameters of one diode of this bridge are given in Table 1. Most useful datasheet parameters of single diode of the bridge GBPC1510 are as follows: $V_{F} = 1.23$ V, $V_{(T0)} = 0.78$V, $r_{T} = 55$ mΩ, $I_{FAV} = 15$ A, $I_{FSM} - 300$ A.
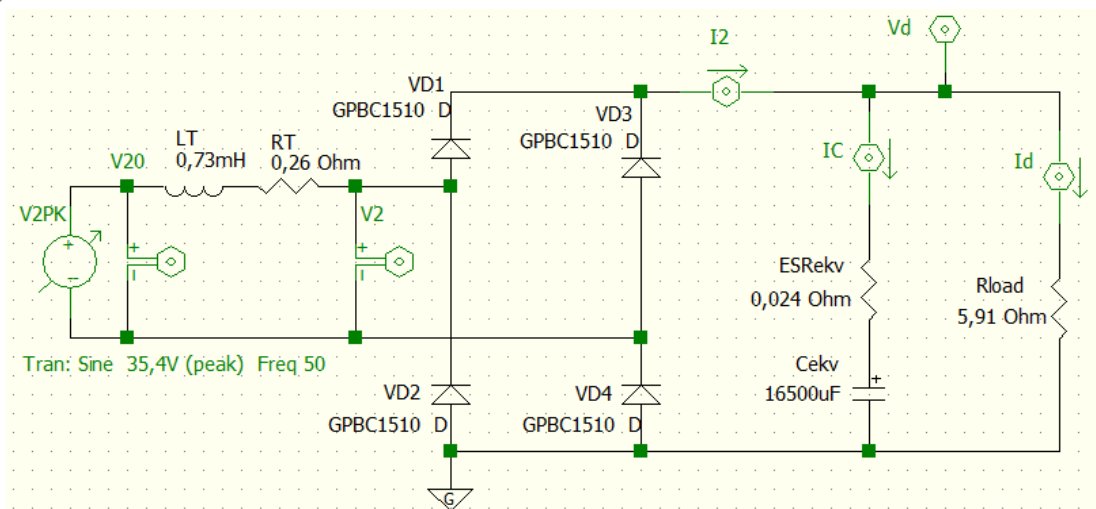

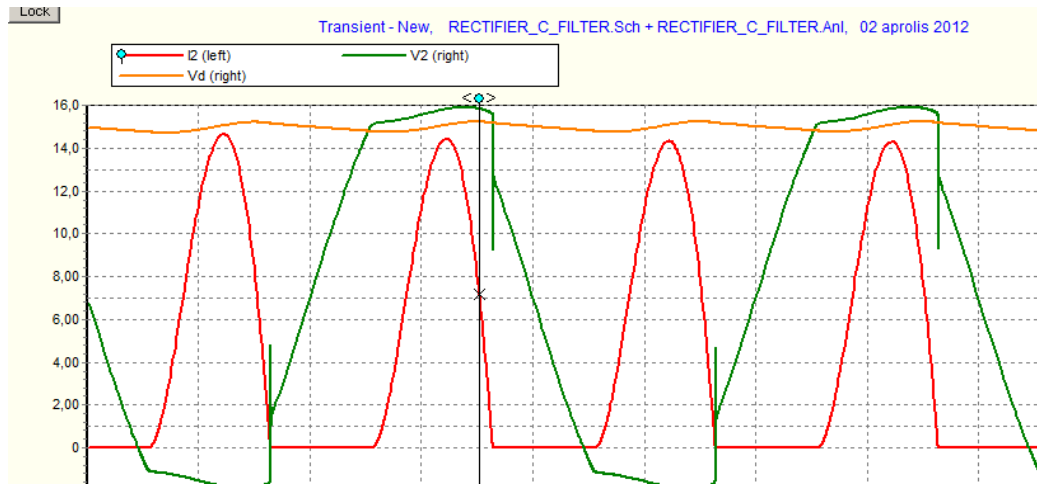Fig. 1. **Circuit schematic in 5SPICE circuit editor**

Fig. 2. **SPICE simulation graph**

Table 1

Diode SPICE parameters

Parameter	Unit	SKN20	GBPC1510
IS	A	170n	6.5n
RS	Ω	0.009	0.028
N		2	1.45
EG	eV	1.55	1.11
CJO	F	400p	300p
VJ	V	1	1
M		0.35	0.494
FC		0.5	0.5
TT	s	10n	10n
BV	V	400	1000
IBV	A	4m	5u
AF		1	1
KF		0	0
XT1		3	3

Excel spreadsheet with the corresponding values is visible in Figure 3 where the green cells contain the given values; gray cells are manually operable ones while the yellow cells display key values for comparison. Calculation result comparison between Excel algorithm and SPICE simulation is given in Table 2.

Table 2

Result comparison between Experiment, Excel algorithm and SPICE model

Parameter	Unit	Experiment	Excel	SPICE
T_{CI}	ms	5.5	5.0	5.2
V_{d0}	V	33.4	32.9	–
I_{2PK}	A	12.3	15.2	14.3
V_{dAVG}	V	25.7	26.6	–
I_d	A	4.35	4.5	–
T_{CU}	ms	4.4	4.7	4
T_{DU}	ms	5.6	5.3	6
ΔV_d	V	1.45	1.6	1.50
I_{2RMS}	A	6.6	7.8	–
I_{2AVG}	A	4.3	5.0	–
ΔP_{VD4}	W	13	14.4	–
V_{2RMS}	V	23.3	23.0	–

Table 2 (continued)

Parameter	Unit	Experiment	Excel	SPICE
I_{CRMS}	A	4.9	6.3	–
V_{dmax}	V	–	27.3	27.3
V_{dmin}	V	–	25.8	25.8
P_d	W	112	120	–
P_2	W	125	135	–

Analyzing the calculation results we can note that significant calculation error (18 %) is introduced in formula (27) – I_{2RMS}. Formula (27) is evaluated on the simplified mathematical model assuming that $\Delta V_d = 0$, besides, the result is calculated using the values of I_{2PK} and I_{2SPK}, which are also calculated through approximated models. The rest or the formulas which are not after I_{2RMS} give the value within 4% tolerance range.

Fig. 3. **Excel spreadsheet example**

Conclusions

As a result of the research new series of formulas suitable for full analysis of the single phase bridge rectifier with a capacitor filter using widely available MS Excel software were introduced. The given series of formulas written into the spreadsheet (without macros) form the semi-automatic analysis algorithm which gives the voltage values within 4 % tolerance range, current values in 18 % tolerance range while the power values in 8 % range.

References

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