MATHEMATICAL MODEL OF INTERACTION OF OPERATING ELEMENT OF TOP REMOVER WITH HEAD OF ROOT CROP

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Abstract. A system of differential equations of the second order has been compiled which describes the movement of a point of the operating element along the head of a sugar beet root rigidly fixed in the soil. On the basis of the developed mathematical model of the interaction of the operating element of the top remover with the head of a root crop the solution of the derived expressions on the PC analytical dependencies of the forces has been obtained at the given point of contact. Graphic dependencies have been built showing how the components of a normal reaction vary at the contact point of the operating element with the head of a root crop. Analytical expression has been determined for the surface area of the head of the coot crop cut off by one operating element.

Key words: sugar beet tops, root crops, operating element, point of contact, force of interaction.

Introduction

High-quality gathering of the sugar beet tops is one of the urgent tasks in the beet growing branch. Removing the remnants of the tops from the heads of the root crops before their extracting from the soil is an important part of the technological process of the sugar beet harvesting. If the share of impurities (the green mass) in the pile of sugar beet roots increases by 1 %, the yield of sugar is reduced by 0.1 %; but, if the root crops are stored in clamps with a 4 % content of the beet tops, then the daily losses of sugar are, on the average, 0.5 % [1; 2]. An insignificant amount of the tops left on the heads of the root crops before their harvesting considerably deteriorates the quality characteristics, which, on the whole, may lower the quality of the product obtained by 10-15 % [3].

There are many treatises by P.Vasilenko, V.Bulgakov, A.Vilde etc. devoted to the problem how to remove the tops from the heads of the root crops [1; 2; 4; 5]. However, these treatises generally deal with the passive top removers, and the obtained dependencies cannot be applied to substantiate the parameters of complex mechanical top removal systems [6; 7].

Materials and methods

Simulation methods are applied in this paper based on the assumptions of theoretical mechanics which characterise the movement of the operating element as a material point. There is investigated the movement of a single operating element along the head of the root crop [8]. During the research process the initial task was solved as a system of nonlinear differential equations of the second order. To detect the forces arising as a result of interaction of the rotary operating element with the head of the root crop, first of all an equivalent scheme (Fig. 1) was drawn up in which the operating element, while performing a rotary and reciprocating movement, is in contact with the head of the sugar beet root [8; 9].

The following forces are active at the point of contact K of the operating element with the head of the root crop:

- $\overline{Q}$ – the hackling force of the beet tops tangentially directed to the surface of the root crop head, towards the vector of absolute velocity of the point M of the operating element;
- $\overline{N}$ – normal reaction from the side of the root crop head directed along normal $\overline{n}$ to the head of the root crop, drawn through the given position of the point of contact;
- $\overline{F}_{тр}$ – the force of friction arising while the operating element is moving along the head of the root crop, directed towards the side opposite to the direction of the vector of absolute velocity of the point M of the operating element and coinciding with the point of contact K, and presented in the form of components on axes x and y; $\overline{G}$ – the force of gravity of the operating element.

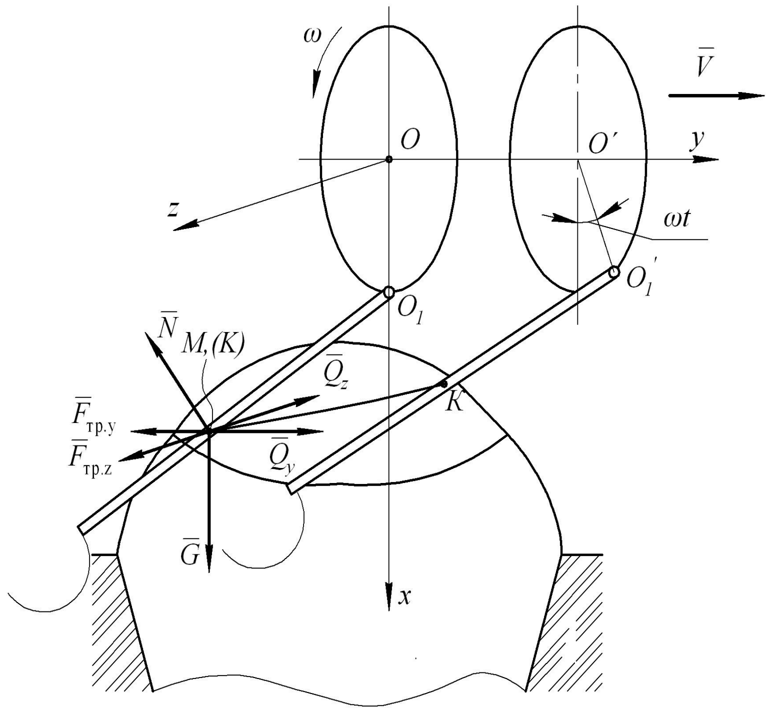

Fig. 1. **Equivalent scheme of interaction of the operating element of the rotary top remover with the head of the sugar beet root**

Results and discussion

The differential equation of the movement of the point of contact *K* along the head of the root crop in a vectorial form will have the following appearance [8]:

$$m\bar{a} = \bar{Q} + \bar{N} + \bar{F}_{\text{тр}} + \bar{G}, \tag{1}$$

where $\bar{a}$ – absolute acceleration of the point of contact *K* along the head of the root crop;
m – mass of the operating element, applied to the point of contact.

In this case the mass of the operating element is applied to the point of contact in order to ensure a possibility to treat its movement as a material point along the surface of the head of the root crop, and to write a differential equation.

After substitution in (1) of the necessary values and transformations there is a system obtained of nonlinear differential equations of the second order related to unknown functions *x*(*t*), *y*(*t*) and *z*(*t*), and the unknown normal reaction *N*:

$$\left.\begin{aligned} m\ddot{x} &= Q\frac{\dot{x}}{V} + N\frac{x}{R} - f\frac{\dot{x}}{V}N + mg, \\ m\ddot{y} &= Q\frac{\dot{y}}{V} + N\frac{y}{R} - f\frac{\dot{y}}{V}N, \\ m\ddot{z} &= Q\frac{\dot{z}}{V} + N\frac{z}{R} - f\frac{\dot{z}}{V}N, \\ x^2 + y^2 &+ z^2 - R^2 = 0. \end{aligned}\right\} . \tag{2}$$

The system of differential equations (2) can be solved only by numerical methods using computer programmes at the given initial conditions. Since the system of differential equations (2) includes an unknown force factor – the normal reaction *N*, its detection was carried out by studying the kinematics of the interaction of the operating element with the head of the root crop. As a result of this study the dependence of the angle of deviation φ of the operating element and its angular acceleration $\ddot{\varphi}$ on time was determined.

Assuming that the head of the sugar beet root is an absolutely solid body and that the operating element is in constant contact with the root crop, a differential equation was developed for the rotary movement of the operating element around its own axis at the moment of its interaction with the head of the root crop [6]:

$$J\ddot{\varphi} + \sum M = 0, \tag{3}$$

where $\sum M$ – sum of the moments of centrifugal forces of inertia acting upon the operating element.

Equation (3) is composed as an additional differential equation to the system of differential equations (2) in order to find the unknown component N – a normal reaction of the operating element to the head of the root crop. The system of equations (2) describes the movement of the contact point of the operating element with the head of the root crop but the differential equation (3) is composed as an additional differential equation for the movement of the whole operating element. It was joint solution of the system of differential equations (2) and the additional differential equation (3) on the PC in the systems Matlab 7.9 and Mathcad 15 using the composed programme VBA that provided a possibility to obtain the value of the normal reaction N.

From equation (3), after algebraic transformations [2; 8], the normal reaction was determined upon the sugar beet root when a system of two operating elements acts upon it:

$$N = \frac{J\ddot{\varphi} + M_R - \dfrac{J\ddot{\varphi}_2 + M_{R2}\sqrt{sin^2\varphi_2 cos^2\alpha + cos^2\varphi_2}\; m_{21}}{\sqrt{\left[\left(r_0 + l_{\ddot{\imath}\ddot{\imath}}\, cos\varphi_2\right)cos\alpha - \dfrac{b}{2}sin\alpha\right]^2 + \left[\left(r_0 + l_{\ddot{\imath}\ddot{\imath}}\, cos\varphi_2\right)sin\alpha - \dfrac{b}{2}cos\alpha\right]^2}}}{\sqrt{\left[\rho\, sin(\alpha_0 + \omega t)\right]^2 + \left[-tg\varphi_0\left(\sqrt{\delta^2 + (d-h)^2 - \left(\dfrac{b}{2}\right)^2} - r_0\right) - Vt\right]^2}}, \tag{4}$$

where δ – deviation of the rotor axis from the conditional axial line of the row of beets; b – designed width of the operating element;
M_R and M_{R2} – moments of centrifugal forces of inertia of the preceding and the successive operating elements relative to their axes of suspension;
m_{21} – normal reaction arm of the coercion of the successive operating element upon the preceding relative to its suspension axis;
φ_2 – deviation angle of the successive operating element from the plane of rotation; α – rotation angle of the rotor shaft;
ρ – distance from the axis of the rotor to the top of the root crop head;
d – distance from the axis of the rotor to the level of the soil surface;
r_0 – radius of the suspension axis of the operating element;
l_{nn} – length of the copying part of the operating element.

The values of the parameters are: $\delta = 0$ m, $b = 0.07$ m, $a_z = 0\ldots360$ (the process is regarded within the limits of a single revolution of the rotor), φ_1, φ_2 – 50…80 degrees, $\rho = 0.2\ldots0.15$ m, d = 0.3 m, $r_0 = 0.3$ m, $l_{nn} = 0.7$ m.

While moving along the head of the root crop, the operating element can act upon it in a longitudinal direction of the row of beets as well as in a transversal direction, pressing also the head of the root crop in a vertical direction. This takes place because the force factors are projections of the total normal reaction to the directions mentioned. By the way, the component N_z of the normal reaction causes knocking of the root crops in a transversal direction and the component N_y – in a longitudinal direction relative to the row of beets. But the vertical component N_x of the normal reaction ensures the necessary force in the contact "the head of the root crop – the operating element". These forces affect such quality characteristics of the top removal process as damage and knocking out of the root crop heads. They have the following values:

$$N_X = \left[\frac{\gamma s l \omega^2 \left(\frac{1}{2} r_0 \sin\varphi + \frac{l^2}{6} \sin 2\varphi \right)}{OK} + \frac{\ddot{\varphi}}{OK} \right] \cos\omega t \times$$

$$\times \sin\left[\varphi_0 + arcctg\left(\frac{-tg\varphi_0 \left(\sqrt{\delta^2 + (d-h)^2 - \left(\frac{b}{2}\right)^2} - r_0 \right) - Vt}{\sqrt{\delta^2 + (d-h)^2} \sin\left[\omega t - \arccos\left(\frac{b}{2\sqrt{\delta^2 + (d-h)^2}} \right) \right] - r_0} \right) \right], \quad (5)$$

$$N_Y = \left[\frac{\gamma s l \omega^2 \left(\frac{1}{2} r_0 \sin\varphi + \frac{l^2}{6} \sin 2\varphi \right)}{OK} + \frac{\ddot{\varphi}}{OK} \right] \times$$

$$\times \cos\left[\varphi_0 + arcctg\left(\frac{-tg\varphi_0 \left(\sqrt{\delta^2 + (d-h)^2 - \left(\frac{b}{2}\right)^2} - r_0 \right) - Vt}{\sqrt{\delta^2 + (d-h)^2} \sin\left[\omega t - \arccos\left(\frac{b}{2\sqrt{\delta^2 + (d-h)^2}} \right) \right] - r_0} \right) \right] , \quad (6)$$

$$N_Z = \left[\frac{\gamma s l \omega^2 \left(\frac{1}{2} r_0 \sin\varphi + \frac{l^2}{6} \sin 2\varphi \right)}{OK} + \frac{\ddot{\varphi}}{OK} \right] \times$$

$$\times \sin\left[\varphi_0 + arcctg\left(\frac{-tg\varphi_0 \left(\sqrt{\delta^2 + (d-h)^2 - \left(\frac{b}{2}\right)^2} - r_0 \right) - Vt}{\sqrt{\delta^2 + (d-h)^2} \sin\left[\omega t - \arccos\left(\frac{b}{2\sqrt{\delta^2 + (d-h)^2}} \right) \right] - r_0} \right) \right] , \quad (7)$$

where N_x – vertical component of the normal reaction which will deform the head of the root crop and press the root crop into the depth of the soil;
N_y – horizontal component of the normal reaction which will knock the root crop in the direction in which the machine is moving;
N_z – horizontal component of the normal reaction which will knock the root crop in the direction perpendicular to the axis of the rotor.

By solving the system of equations (2) relative to the unknown functions $x(t)$, $y(t)$, $z(t)$ we obtain an equation in a parametric form of the path of the contact point K moving along the head of the root crop:

$$\left. \begin{aligned} x &= x(t), \\ y &= y(t), \\ z &= z(t). \end{aligned} \right\} \quad (8)$$

The length of the spatial curve is expressed as:

$$L = \int_{0}^{t_1} \sqrt{\dot{x} + \dot{y} + \dot{z}^2}\, dt, \tag{9}$$

where t_1 – duration of the contact of the cutting part of the top remover with the head of the root crop.

In this case the area of the cutting surface of the tops $S_{зч}$ will be equal to:

$$S_{зч} = Lb. \tag{10}$$

Using the results of numerical simulation on the PC, graphic dependencies of time were built (Fig. 2-4) for the indicated component forces. The duration of the contact of a single operating element with the root crop is $3 \cdot 10^{-3}$ s [2]. The entire process of interaction with the system of elements goes on for about $20\text{-}25 \cdot 10^{-3}$ s [10]. The graphs represent the period of time of interaction before the 3rd element enters the system.

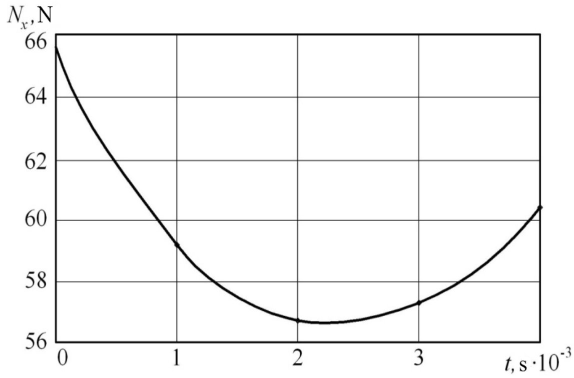

Fig. 2. **Dependence of the longitudinal horizontal component N_x of the normal reaction on time**

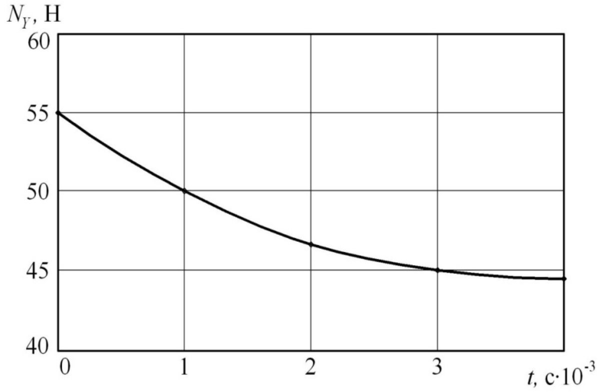

Fig. 3. **Dependence of the horizontal component N_z of the normal reaction on time**

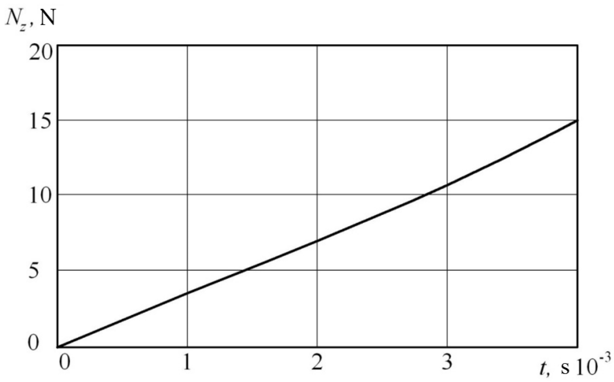

Fig. 4. **Dependence of the transversal horizontal component N_z of the normal reaction on time**

Conclusions

1. A system of differential equations of the second order has been compiled describing the movement of a contact point of the rotary operating element along the head of a sugar beet root rigidly fixed in the soil.
2. By solving a differential equation of the rotary movement of the operating element an analytical expression has been found for the normal reaction of the head of the root crop to the operating element.
3. By solving the obtained system of equations an analytical expression has been found which allows determining the surface area of the head of the root crop removed by a single operating element.

4. On the basis of the obtained model maximal values have been determined for the normal reactions acting at the contact point of the operating element with the head of the root crop.

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