RESEARCH OF BUCKLING STABILITY OF FLAT-TYPE MULTILAYER RUBBER-METAL PACKAGE

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Abstract. Packages of thin-layered rubber-metal elements (TRME) are successfully used as bearings, joints, compensating devices, vibration and shock absorbers etc. TRME significantly outperform the traditional elements of the same purpose because of their reliability, low cost, simplicity of structural layout and assembly. TRME packages usually work under heavy compressive loads. TRME have high compression stiffness, which are several orders of magnitude greater than their shear stiffness. With the compressive forces increasing shear stiffness of such packets is reduced which leads to a loss of buckling stability. For TRME package under compression, unlike the classical theory of rod buckling stability, the shear instability takes place. In this paper bucling of flat-type TRME packages of rectangular shape under compression is discussed. The next formulas are derived for package design: the dependence of the critical external loads on loading conditions, on packages end-fixity conditions, on layers geometrical parameters and mechanical properties of layers materials. The dependence of mechanical modules of elastomeric on the compressive load level is taken into account. It is assumed that the non-elastomeric layers are either perfectly rigid, or may undergo only a plane tensile strain. Small deformation is considered. The solutions obtained are compared with experimental data of other authors.

Keywords: elastomeric, multilayer devices, stiffness, buckling, variational method.

Introduction

Elastomers (natural and synthetic rubber) are a unique family of materials which offer many engineering advantages because of their small volume compressibility and the ability to maintain large elastic deformation [1-3]. Reinforced elastomeric structures (or laminated elastomer) consist of alternating thin layers of rubber and adhesive-bonded reinforcing layers of a much more rigid material (usually metal). This allows to obtain the structures, which axial compression stiffness is in several orders greater than shear stiffness. Packages of thin-layered rubber-metal elements (hereinafter referred to asTRME) are successfully used as bearings, joints, compensating devices, shock-absorbers etc. [1-4] In practice TRME packages of different geometrical form are used: flat of various shape, cylindrical, conical etc; the number of layers may be different, at least three (Fig. 1).

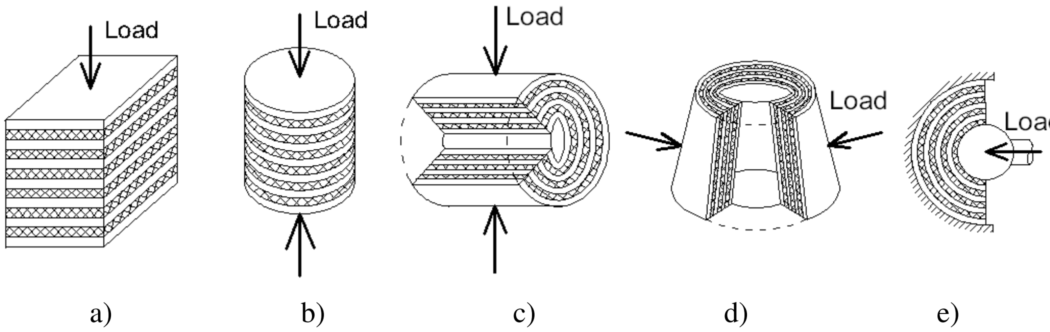


Fig. 1. **Multilayer elastomeric structures examples:** a – flat rectangular; b – flat circular; c – cylindrical; d – conical; e – spherical

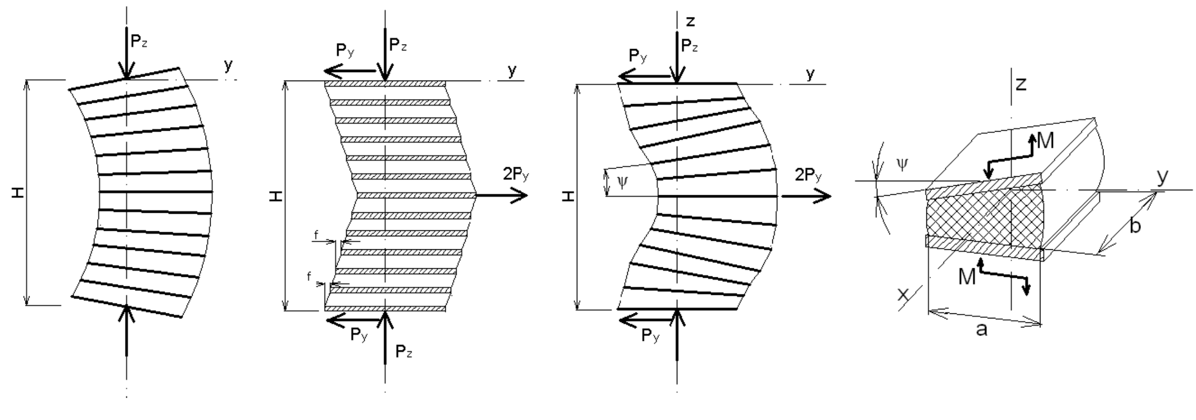
Elastomeric layer is considered as thin if its width/thickness ratio is much more than ten. Multilayered packets of thin-layer rubber-metal elements in which the ratio $\alpha = a/h_e > 10$ and $\beta = b/h_e > 10$ (where a and b – the dimensions of the elastomeric layer in the plane, h_e – thickness of the elastomer layer) have extensive use almost in all spheres of engineering and construction (joints and bearings for various applications, support of engineering structures, vibration and shock absorbers ets.). In such packages working under significant compressive loads the buckling of the middle layers of packet is observed, i.e., the package loses buckling stability, which leads to decreasing of performance capabilities of packages and their failure. Buckling has shear instability form (the layers are shifted sidewise), rather than bending (as in the classical theory of rods stability). This occurs because of TRME stiffness under axial compression and the bending stiffness is in several orders greater than the shear stiffness.

Gent A. N. considers the stability of structures with thick rubber layers (with the shape factor ≈1) based on the classical theory of rods. This approach and the main position of Gent's work was later used by many authors [3; 5; 6], but further investigations show that application of these solutions to thin rubber-metal elements leads to significant errors [3; 7]. Many successive works deal with TRME package buckling stability [8-11], the method of bending stiffness calculation on the assumption that the middle surface of elastomeric layer remains flat under deformation was elaborated [10]. The works [12; 13] where the discrete analysis of TRME package stability in matrix form was developed, are the most comprehensive. Nevertheless, practical use of the obtained results is limited due to the lack of technics for definition of elastomeric layer stiffness coefficients included in the matrix form of constitutive equations. An additional point is conection of TRME packages buckling stability with assembling errors: parallel misalingnment of nonelastomeric layers and wedging of elastomeric, inaccuracy of layers manufactiring and so on; this question should be discussed separately.

When designing TRME packages to improve their operational characteristics and increase the permissible rate of compressive loads, it is necessary to have an analytical expression (preferably in a simple manner) to calculate the critical external load taking into account the TRME geometric parameters, scheme of external load imposing and method of TRME packet fastening, mechanical properties of materials.

Materials and methods

In the mentioned above studies it is assumed that: nonelastomeric layers are nondeformable, external forces are conservative, elastomeric and nonelastomeric layers are rigidly connected to each other, the deformation of each individual TRME is linear. Besides that, the assumptions are introduced that the elastomeric material layer is volumetrical incompressible and its mechanical properties are not dependent on the rate of external loading. But there is not argumentation of the above listed assumptions applicability domain and estimation of their influence on the numerical value of the critical forces. In the given paper the methodology of calculating the critical force for TRME package buckling taking into account the weak compressibility of elastomeric layers and shear modulus dependence on the load level which were not considered in the works [3; 9-11] is discussed. As an example the stability problem of the rectangular – type flat TRME package under axial compression between two flat parallel plates by the force P is considered. In Fig. 2 the forms of loss of stability of the TRME device under axial compression are shown; because the compression and tension stiffness are much greater than the share stiffness, the considering buckled shape includes share deformation (Fig. 2c).



a) b) c) d) e)

Fig. 2. **Loss of stability of TRME device under axial compression:** a – Euler buckling; b – pure shear buckling; c – Euler buckling with share contribution; d – scheme of bending for TRME section

The TRME package (of thickness $H = h_c N$) consists of N individual identical sections. Each section (thickness $h_c = h_e + h_m$) consists of a nondeformable metal plate (thickness – h_m) and is vulcanised to an elastomeric layer (thickness – h_e and sectional area – F), which deformation is considered as small. When calculating the shear stiffness of the elastomeric element K_y of the shear force P_y the scheme of simple share is applied; for bending stiffness T calculating – the scheme when the metal plates are rotated with respect to each other relative to the axis of symmetry.

For the given loading scheme (Fig. 1) lateral deviation y_{max} for the section in the central part of the package (z = 0.5 H) is found from the derived in [3] equation:

$$\frac{K_{y_{\max}} h_c}{P_y(0.5H)} = \frac{K_y h_c}{P_z}\left[\left(1+\frac{P_z}{K_y h_c}\right)\frac{tg(0.25qH)}{0.25qH}-1\right],\ q^2 = \frac{P_z}{Th_c}\left(1+\frac{P_z}{K_y h_c}\right), \tag{1}$$

where q – function of the axial compressive force.

From (1) at $0.25qH \to 0.5\pi$ lateral displacement increases infinitely and the buckling condition may be written as:

$$\frac{P_{z,cr}}{Th_c}\left(1+\frac{P_{z,cr}}{K_y h_c}\right) = \frac{4\pi^2}{H^2} \tag{2}$$

Critical value $P_{z,cr}$ axial compressive force is from the stability condition (2):

$$P_{z,cr} = 0.5K_y h_c\left(\sqrt{1+\frac{16\pi^2 T}{K_y H^2}}-1\right). \tag{3}$$

Shear stiffness of the elastomeric layer K_y determined from pure shear scheme is:

$$K_y = \frac{GF}{h_e}. \tag{4}$$

Bending stiffness without accounting of the elastomeric layer low compressibility:

$$T = \frac{2(1+\mu)GI_x\left(1+\gamma\Phi^2\right)}{h_e},\ \Phi = \frac{F_l}{F_f} = \frac{ab}{h_e 2(a+b)}\ , \tag{5}$$

where G – shear modulus of the elastomer;
I_x – axial moment of inertia of the cross section of the element of the TRME package;
Φ – shape factor;
γ – empirical coefficient
F_l – loaded surface area of the block;
F_f – free surface area.

The dependence (3) ÷ (5) gives acceptable results for elastomeric layers with a shape factor 1 ÷ 2 (or b/h_e and a/h_e < 5), small deformations and for specific axial load P_z/F to 5 ÷ 10 MPa [4]. To determine the critical axial compression force for small deformations domain, thin layers (b/h_e and a/h_e> 10) and high specific axial loads (P_z/F > 10 MPa) formula (3) may be used if instead stiffness (4) and (5) to substitute shear K_y and bending T stiffness calculating with accounting of the elastomeric layer weak compressibility and the loading level effect on shear modulus of the elastomeric material.

For thin layers one elastomeric layer bending stiffness T (load scheme is given in Fig. 2) is calculated, using the principle of minimum of total potential energy of deformation [14] subject to the weak compressibility of elastomer, assuming that metal layers are nondeformable:

$$\Pi = G\int_V \left[\frac{1}{2}\left(u_{i,j}u_{i,j}+u_{i,j}u_{j,i}\right)+\frac{3\mu}{1+\mu}su_{i,i}-\frac{9(1-2\mu)}{4(1+\mu)^2}s^2\right]dV - M\psi, \tag{6}$$

where s – spesific hydrostatic pressure;
u_i – axial displacement functions;
V– volume of elastomeric layer;
i, j – coordinates x, y, z of cartesian coordinate system, over repeated subscripts summation is fulfilled, a comma in the subscript denotes the partial derivative.

For a single TRME layer the origin of coordinate is choosen in the center of gravity of the element (Fig. 2). Using the functional (6) requires the mandatory implementation of the geometrical boundary conditions for the displacements:

$$u_x(x, y, \pm 0.5h_e) = 0;\ u_y(x, y, \pm 0.5h_e) = 0;\ u_z(x, y, \pm 0.5h_e) = \pm y\psi. \tag{7}$$

For the function s (x, y, z) there are no mandatory boundary conditions as forces boundary conditions for the functional (6) are the natural boundary conditions. Considering the conditions (7) the displacements and hidrostatic pressure functions are selected:

$$u_x = A_3 \frac{xy\pi}{h_e}\cos\left(\frac{z\pi}{h_e}\right);\ u_y = A_1 \frac{y^2}{2h_e}\cos\left(\frac{z\pi}{h_e}\right) + A_2 \cos\left(\frac{z\pi}{h_e}\right); \tag{8}$$

$$u_z = A_4 y \sin\left(\frac{z\pi}{h_e}\right);\ s = A_5 y \cos\left(\frac{z\pi}{h_e}\right) + A_6 y.$$

Since from the equations (7) and (8)

$$\psi = \frac{u_z(x, y, \pm 0.5h_e)}{y} = A_4$$

the dependence "M-ψ" is found by the constant A_4 without defining other constants of the system from the condition of functional (6) minimum:

$$\frac{\partial \Pi(A_1, A_2, A_3, A_4, A_5, A_6)}{\partial(A_1, A_2, A_3, A_4, A_5, A_6)} = 0. \tag{9}$$

Since a significant effect of weak compressibility (depending on the Poisson ratio value) may be expected only for sufficiently thin elastomeric layers, the system (9) may be simplified, leaving only the factors and terms proportional to α^2 and β^2.

In this case for the middle section of thin elastomeric layers package, dependence "bending moment – rotation angle" (M-ψ) and the bending stiffness T_1 may be written, considering the weak compressibility of the elastomer:

$$\psi = \frac{M}{T_1};\ T_1 = \frac{Ga^3\beta D}{1 + 6(1 - 2\mu)D};\ D = \frac{0.125(\alpha^2 + \beta^2)}{\alpha + \beta};\ \alpha = \frac{a}{h_e} > 10;\ \beta = \frac{b}{h_e} > 10. \tag{10}$$

In the considered case, if the change of thickness of the elastomeric layer is not taken into account, the shear stiffness K_y does not change and may be calculated by the formula (4) for the scheme of pure shear. Buckling load $P_{z,cr}$ is calculated by the formula (3), substituting the bending stiffness T_1 for T.

The results of the experiments on thin TRME compression [3; 14] show that at relatively small strains (up to 10 ÷ 15 %), specific loading (P_z/F) may reach 200 MPa. The dependence of the "force – displacement" has a highly nonlinear character, indicating that the mechanical modules of elastomer depend on the level of the specific compressive strength even in small deformation area. In experimental studies it is shown that shear and bulk modulus of elastomeric layer G and K depend on the intensity of the specific loading if $s = P_z/F$ is more than 5 MPa [14; 15].

In order to take into account load intensity influence on "force – displacement" dependence the author of work [16] proposes to take the linear solution as an approximate solution, in which instead of modules G and K it is necessary to substitute the values $G(s)$ and $K(s)$ which correspond to the average hydrostatic pressure value s (x, y, z). For thin flat elastomeric layers it can be assumed with sufficient accuracy that $s = P_z/F$ (where F – the area of plane layer). This approximation will be the better, the less distortion -strain energy of a thin elastomeric layer specific contribution into "force – displacement" dependence. This approach allows calculating of $G(s)$ and $K(s)$ from the volumetric "tension – compression" experiments with accuracy up to the assumption of small deformations. Due to lack of experimental data it is proposed in [14; 15] at first approximation to assume that the dependence of *G(s)* and *K(s)* has the same type:

$$G(s)/G \approx K(s)/K = 1 + \varphi \cdot s\,, \tag{11}$$

where the factor φ is defined from the experiment on pure volumetric compression. Therefore, when designing the stability of TRME package with very thin flat rectangular layers under axial compression between two flat parallel plates (Fig. 2) for calculating the critical force it is recommended to apply the equation:

$$P_{z,cr} = 0.5K_y h_c \left(1 + s\varphi\right)\left(\sqrt{1 + \frac{16\pi^2 T_1}{K_y H^2}} - 1\right). \tag{12}$$

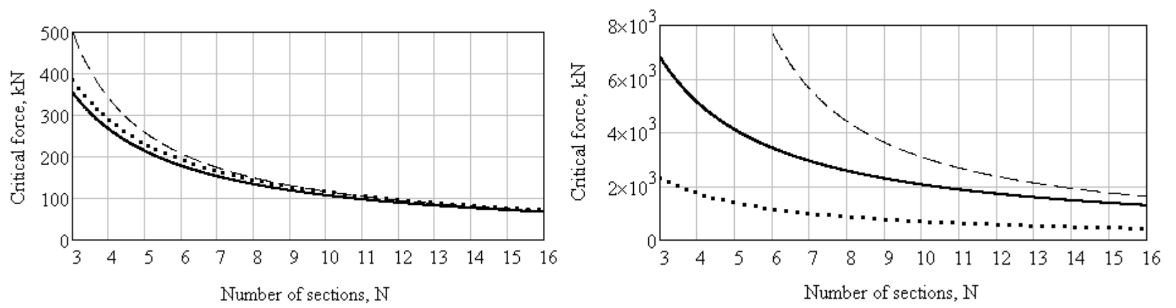
This equation takes into consideration formulas (3), (4), (10), (11) and allows to estimate the critical force for large values of the specific external compressive load.

Results and discussion

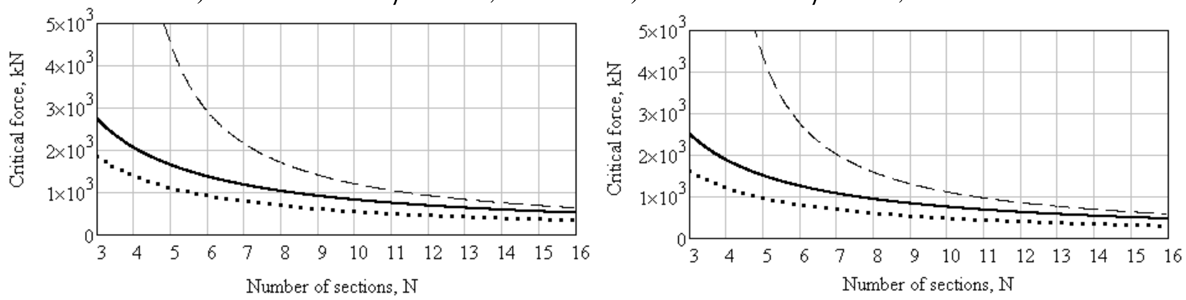
The results of critical forces calculation for flat rectangular TRME package (with typical in industrial application dimensions) are presented bellow. Plots of buckling force dependence on the number of sections in the packet are given for four types of TRME, which were fabricated and tested in the Moscow Institute "Teploprojekt". For thin layered packages the experimental critical force does not coincide with calculating in accordance with conventional equation (3).

In Fig. 3a buckling force plots for TRME packet PRM-210 with steel rigid layers are shown; PRM-210 data: a = 35 mm, b = 35 mm, h_e = 0.44 mm, h_m = 0.1 mm, h_c=0.54mm, shape factor Φ = 19.9, G = 0.45MPa, μ = 0.4981, φ = 0.001. In Fig. 3b buckling force plots for PRM-35 are given; material properties and dimensions are the same as for PRM-210, excluding h_e = 0.1 mm, Φ = 87.5.

In Fig. 3c buckling force plots for sample No.2 with brass bonded layers are given; dimensions: a = 49.5 mm, b = 52.5 mm, h_e = 0.33 mm, h_m = 0.05 mm, h_c = 0.38 mm, Φ = 38.6, G = 0.45MPa, μ = 0.4981, φ = 0.001. In Fig. 3d buckling force plots for sample No.6 are shown; No.6 data: a = 47 mm, b = 49 mm, h_e = 0.58 mm, h_m = 0.05 mm, h_c= 0.63mm, Φ = 20.6, G = 1.6MPa, μ = 0.4903, φ = 0.001.



a) PRM-210: $\alpha = \beta$= 79.5, Φ = 19.9 b) PRM-35: $\alpha = \beta$ =350, Φ=87.5



c) No.2: α =150, β = 159, Φ = 38.6 d) No.6: α =81, β= 84, Φ = 20.6

Fig. 3. **Plots of critical force dependence on number of TRME sections device under axial compression:** ——— in accordance with equation (3), ... in accordance with (3) taking into account (10), — –in accordance with equation (12)

The specific load in all cases is more than 20 MPa. It is seen from the plots how the critical force value depends on the thickness of elastomeric layers and on the number of the sections.

Conclusions

This work presented the methodology of the buckling force calculation for thin-layered rubber-metal packages widely used as vibroisolators, shock absorbers, and compensation devices. Such devices usually carry very large load and should be checked on buckling. Three approaches are discussed:

1. conventional;
2. taking into account the thickness of elastomeric layers;
3. taking into account the thickness of elastomeric layers and changing of the elastomeric mechanical properties (shear and bulk modules) depending on pressure.

The results of the numerical examples show that at the number of layers in the package increasing the critical force value becames closer. Each type of TRME demands individual approach.

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