ESTIMATED MATHEMATICAL MODEL OF PLANE-PARALLEL MOTION OF TRAILED HEMP HARVESTING AGGREGATE

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Abstract. A mathematical model of a trailed hemp harvesting aggregate was built. For this purpose an equivalent scheme of a machine aggregate was worked out at first as consisting of an aggregating wheeled tractor, a trailed hemp harvesting machine and a tractor trailer. As initial equations of dynamics, the Lagrange equations of the 2nd kind were accepted. After that expressions for kinetic energy of the mechanical system and generalised forces were determined considering the forces which act upon the driven and the driving wheels of the tractor, the wheels of the trailed hemp harvesting machine, the trailer, as well as the working resistance caused during hemp harvesting. After the necessary transformations connected with the application of differential equations of motion, a system was obtained consisting of six differential equations of motion. After a programme of numerical calculations was created, their solution was performed on PC, which allowed optimisation of the structural and kinematic parameters of the trailed hemp harvesting machine, ensuring overall stability of its movement during the technological process of hemp harvesting.

Key words: hemp, trailed hemp harvesting machine, equivalent scheme, differential equations of motion.

Introduction

There is a tendency observed in the world during the last two decades towards an active growth in the areas under cultivated industrial hemp. Moreover, there is strict control imposed in the EU countries on the production of this crop, and only those sorts are allowed for cultivation and dotated which contain not more than 0.2% of tetrahidrokanabinol (THC), which is a psychoactive ingredient [1]. In the EU countries industrial hemp (*Cannabis sativa* L.) is considered as one of important renewable resources for the production of a wide range of industrial products (which is witnessed by the Eurocommission Resolution COM/2008/03/07). Major producers of hemp in Europe are France, the Netherlands, Germany, Ukraine, Lithuania. In Latvia massive cultivation of industrial hemp has taken place since 2009 [2].

The most complicated and labour-consuming process in the industrial hemp cultivation technology is its harvesting. The climatic conditions in Eastern Europe are essentially different from those in Western Europe by the vegetation times and harvesting conditions. Considering the fact that massive cultivation of industrial hemp in East European countries has been carried out since a relatively recent time, there is still intense search going on for the most efficient harvesting technologies and corresponding machines for their implementation. Due to the circumstance that the self-propelled hemp combine harvesters used in large numbers in Western Europe have a high price (about 300 thousand EUR), it is supposed to use cheaper trailed hemp harvesting machines in the first hemp production stage in a series of the East European countries (Latvia, Ukraine, etc.) with limited possibilities to invest into machines [3; 4]. Elaboration of the designs of the trailed machines and their rational application is an urgent task for science and technology. Peculiarities of the machine and tractor aggregates with such machines are application of several trailed elements (a harvester, a pickup trailer adapted to receive part of the yield), a relatively complex path of their motion and the great total length. Therefore, analytical research of the motion of complex hemp harvesting aggregates is a topical task of science and technology. The aim of the present work is to develop an estimated mathematical model of the motion of a trailed hemp harvesting aggregate in a horizontal plane considering its design and kinematic parameters.

Materials and methods

A methodology laid out in scientific works [5-7] was used to build mathematical models of complex agricultural machines and machine aggregates. Besides, it was assumed that the basic kind of movement of the complex harvesting machine aggregates is their plane-parallel motion since it is a kind of motion which determines the energy costs, the quality of the performed harvesting processes, etc.

The basis of the methodology mentioned is an initially worked out equivalent scheme of a plane-parallel motion, determination of the forces applied to the component parts of the aggregate, the choice of generalised coordinates which determine the position of the components of the machine aggregate, and acceptance of a coordinate system on their basis, application of the initial equations providing a possibility to describe the behaviour of the dynamic system to be discussed.

The adopted law will allow sufficiently exact description of the behaviour of the dynamic system considered (for the complex agricultural machine aggregates these are, as a rule, the initial equations in the form of the Lagrange equations of the 2nd kind) and execution of operations for the generation of differential equations of motion to be transformed later into a form allowing their numerical solution, and application of the results of this solution for the analysis and synthesis of their design and kinematic parameters.

In the course of analytical research of the plane-parallel motion of the trailed hemp harvesting aggregate we worked out its equivalent scheme (Figure 1.) consisting of an aggregating wheeled tractor 1, a trailed hemp harvesting machine 2, a hitch mechanism 3, and a tractor trailer 4. It was assumed that all the points of the considered dynamic system are moving in planes which are parallel to a certain director plane, i.e. they are performing the plane-parallel motion.

Let us relate the present dynamic system to a fixed system of coordinates *xOyz*. Let us put the axes *Ox* and *Oy* onto the horizontal plane (the surface plane of the hemp field) directing the axis *Oz* vertically up.

Let us admit that in case when the given trailed machine aggregate is moving along the surface of the soil, all its points are moving in the planes which are parallel to the plane *xOy*.

In order to generate differential equations of the motion of the present dynamic system, let us consider it in an intermediate position, and this position will be described by six independent generalised coordinates x_1, y_1, β_1, β_2, β_3, β_4, which uniquely determine the position of this system. Here: x_1, y_1 – the coordinates of the mass centre of the aggregating tractor, β_1, β_2, β_3, β_4, – the corresponding angles formed by the longitudinal axes of the components of the hemp harvesting machine aggregate with the axis *Ox*. In addition, let us introduce such parameters and designations of the present mechanical system:

- $m_i\left(i=\overline{1,4}\right)$ – mass of the components of the machine aggregate;
- $C_i\left(x_i, y_i\right)$ – mass centres of the components of the aggregate;
- a_i – distances from the mass centres of the components to the frontal joint;
- l_i – distances between two adjacent axes of the joints.

Suppose that in the initial moment (t = 0) the given machine aggregate is oriented in an arbitrary position along the axis Ox starting motion from the state of rest.

In order to investigate the motion of a trailed machine aggregate as a complex dynamic system, we will use the initial equations in the form of the Lagrange equations of the 2nd kind [2; 8]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right)-\frac{\partial T}{\partial q_s}=Q_s \quad \left(s=\overline{1,6}\right), \tag{1}$$

where T – kinetic energy of the machine aggregate;
q_s – generalised coordinate;
s – number of the generalised coordinate;
Q_s – generalised force corresponding to the generalised coordinate q_s.

We will calculate the kinetic energy of the machine aggregate as the sum of kinetic energies of all component parts of the aggregate, namely:

$$T=\sum_{i=1}^{4}T_i=\frac{1}{2}\sum_{i=1}^{4}\left[m_i\left(\dot{x}_i^2+\dot{y}_i^2\right)+I_i\omega_i^2\right], \tag{2}$$

where I_i – inertia moment of the i-th component of the aggregate in relation to the vertical axis passing through its mass centre;

$\omega_i = \dot{\beta}_i$ – angular velocity of rotation of the i-th component of the aggregate;

$\dot{x}_i$, $\dot{y}_i$ – projection of the velocity vector of the mass centre of the i-th component of the aggregate.

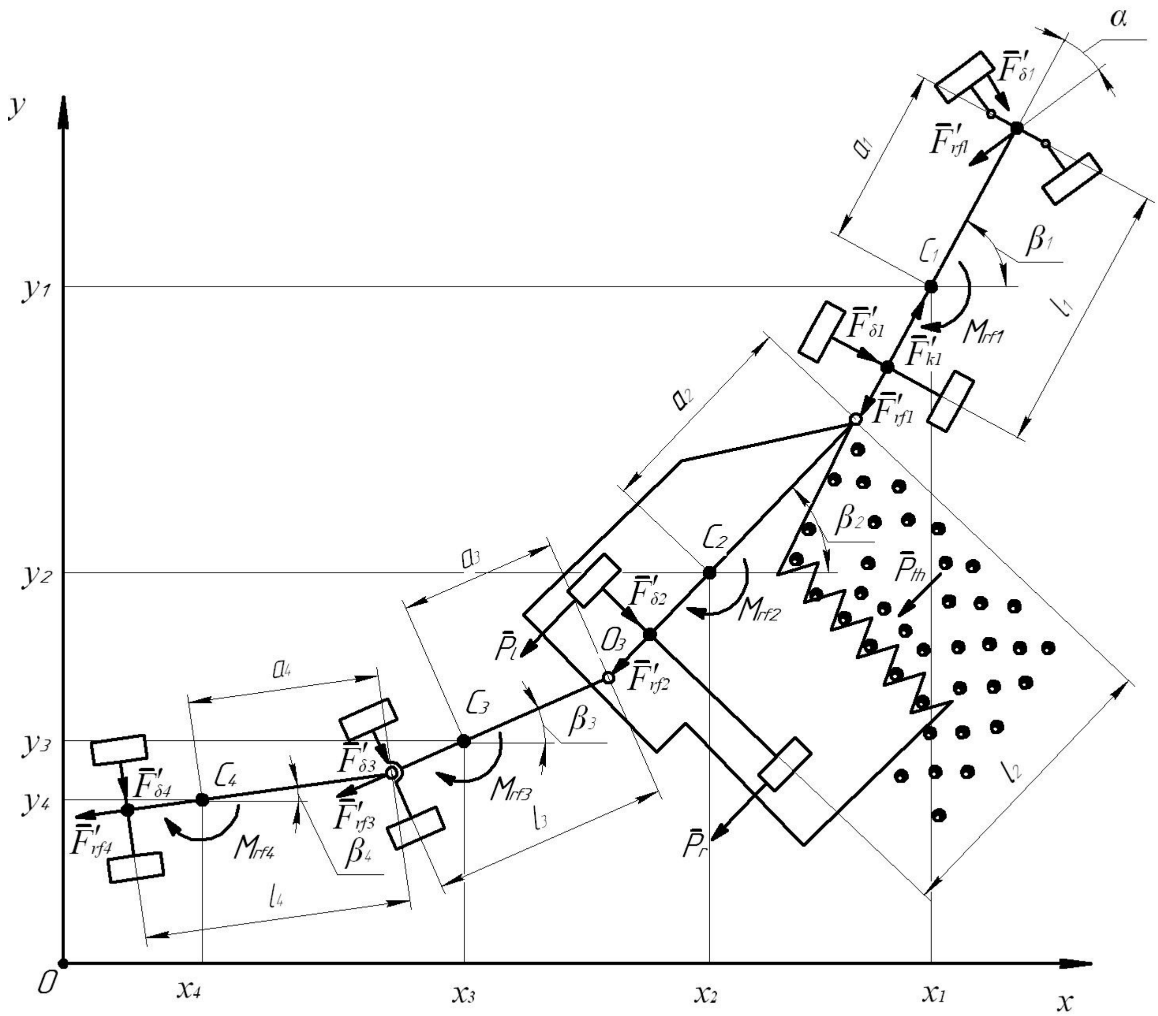

Fig. 1. **Equivalent scheme of the trailed hemp harvesting aggregate:** 1 – tractor; 2 – hemp combine harvester; 3 – hitch mechanism; 4 – trailer

If we designate the coordinates of the mass centre as x_1, y_1 then the coordinates of the mass centres of the i-th $\left(i = \overline{2,4}\right)$ component of the given machine aggregate will be defined by the following expressions:

$$\left.\begin{aligned} x_i &= x_1 - (l_1 - a_1)\cos\beta_1 - a_i\cos\beta_i - \sum_{j=2}^{i-1} l_j \cos\beta_j, \\ y_i &= y_1 - (l_1 - a_1)\sin\beta_1 - a_i\sin\beta_i - \sum_{j=2}^{i-1} l_j \sin\beta_j, \left(i = \overline{2,4}\right). \end{aligned}\right\} \tag{3}$$

Taking the time derivative from the expressions (3), we will determine the velocities of the mass centres of the components of the machine aggregate in projections onto the axes Ox and Oy :

$$\left.\begin{aligned}\dot{x}_i &= \dot{x}_1 + (l_1 - a_1)\dot{\beta}_1 \sin\beta_1 + a_i\dot{\beta}_i \sin\beta_i + \sum_{j=2}^{i-1} l_j\dot{\beta}_j \sin\beta_j,\\ \dot{y}_i &= \dot{y}_1 - (l_1 - a_1)\dot{\beta}_1 \cos\beta_1 - a_i\dot{\beta}_i \cos\beta_i - \sum_{j=2}^{i-1} l_j\dot{\beta}_j \cos\beta_j,\ \left(i = \overline{2,4}\right).\end{aligned}\right\} \tag{4}$$

By substituting expressions (7) into the expression (2) we will obtain an equation of kinetic energy of the given hemp harvesting aggregate:

$$T = \frac{1}{2}\left[m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right) + I_1\dot{\beta}_1^2 + \sum_{i=2}^{4}\left(m_i\left(\dot{x}_i^2 + \dot{y}_i^2\right) + I_i\dot{\beta}_i^2\right)\right]. \tag{5}$$

Let us calculate further the forces corresponding to the generalised coordinates. Considering the circumstance that the generalised forces are coefficients in the expression of elementary performance of all the forces initiating possible movement of the system, we will determine the active forces for this purpose and possible movements of this dynamic system [8].

With this aim in view, we will transfer the forces acting upon the wheels of the given machine aggregate to the frontal and the rear axles of the tractor. The aggregating wheeled tractor has a wheeled formula 4К2 with the rear wheels being driven but its turning being effectuated by changing the position of the frontal (steerable) wheels – turning them by the angle *α*. We will designate these forces as:

- F'_{k1} – draft force of the aggregating tractor;
- F_{rfi} – resistance force of the i-th component of the aggregate;
- $F_{\delta i}$ – lateral force acting upon the i-th component of the aggregate;
- M_{rfi} – resistance moment to the turning of the i-th component of the aggregate;
- P_{th} – resistance force to the motion of the hemp harvesting aggregate caused by pulling of the hemp stalks;
- P_l and P_r – resistance forces to the rolling motion of the wheels applied to the rear axle and included into force F_{rf2} and moment M_{rf2}.

All the force factors mentioned are determined according to analytical expressions presented in [9-10] and in accordance with the experimental data and the test results depending on the type of soil, the parameters of the wheels, the type of the tractor, and so on.

According to the accepted designations, the generalised forces are determined in the following way:

$$Q_{x1} = \sum_{i=1}^{4} F_{xi}, \tag{6}$$

$$Q_{y1} = \sum_{i=1}^{4} F_{yi}, \tag{7}$$

where F_{xi}, F_{yi} – sum of the projections of all forces acting upon the *i*-th component of the machine aggregate on the axes *Ox* and *Oy*, accordingly.

The generalised forces corresponding to the turning angles β_i $(i = \overline{1,4})$ are determined on the basis of analytical determination of the joint coordinates $O_i\left(x_{O_i}, y_{O_i}\right)$, $\left(i = \overline{2,4}\right)$:

$$\left.\begin{aligned} x_{O_i} &= x_1 - (l_1 - a_1)\cos\beta_1 - \sum_{j=2}^{i-1} l_j \cos\beta_j, \\ y_{O_i} &= y_1 - (l_1 - a_1)\sin\beta_1 - \sum_{j=2}^{i-1} l_j \sin\beta_j. \end{aligned}\right\} \tag{8}$$

and their possible movements:

$$\left.\begin{aligned} \delta x_{O_i} &= \delta x_1 + (l_1 - a_1)\sin\beta_1 \cdot \delta\beta_1 + \sum_{j=2}^{i-1} l_j \sin\beta_j \cdot \delta\beta_j, \\ \delta y_{O_i} &= \delta y_1 - (l_1 - a_1)\cos\beta_1 \cdot \delta\beta_1 - \sum_{j=2}^{i-1} l_j \cos\beta_j \cdot \delta\beta_j, \end{aligned}\right\} \tag{9}$$

After summing up the elementary performances of the active forces on the possible angular movements of the system we obtain the following final expression for the generalised forces corresponding to the angular (β_i) generalised coordinates:

$$Q_{\beta_i} = M_{O_i} - M_{rfi} + l_i\left(\sin\beta_i \sum_{j=i+1}^{4} F_{xj} - \cos\beta_i \sum_{j=i+1}^{4} F_{yj}\right), \quad \left(i = \overline{2,4}\right), \tag{10}$$

where M_{Oi} – algebraic sum of the moments of all forces that act upon the i-th component relative to point O_i.

Results and discussion

After the necessary derivatives from the expressions of kinetic energy have been determined, which enter into the initial Lagrange expression of the 2nd kind, and substitution of the obtained derivatives into expression (1), we obtain the following system of differential equations describing the motion of the trailed hemp harvesting aggregate in a horizontal plane:

$$\left.\begin{aligned} & m_1\ddot{x}_1 + \sum_{i=2}^{4} m_i\ddot{x}_i = \sum_{i=1}^{4} F_{xi}, \\ & m_1\ddot{y}_1 + \sum_{i=2}^{4} m_i\ddot{y}_i = \sum_{i=1}^{4} F_{yi}, \\ & I_1\ddot{\beta}_1 + (l_1 - a_1)\sum_{i=2}^{4} m_i\left(\ddot{x}_i \sin\beta_1 - \ddot{y}_i \cos\beta_1\right) = \\ & = M_{C_1} - M_{rf1} + (l_1 - a_1)\left[\sin\beta_1 \sum_{i=2}^{4} F_{xi} - \cos\beta_1 \sum_{i=2}^{4} F_{yi}\right], \\ & I_2\ddot{\beta}_2 + m_2 a_2\left(\ddot{x}_2 \sin\beta_2 - \ddot{y}_2 \cos\beta_2\right) + l_2\left[m_3\left(\ddot{x}_3 \sin\beta_2 - \ddot{y}_j \cos\beta_2\right) + \right. \\ & \left. + m_4\left(\ddot{x}_4 \sin\beta_2 - \ddot{y}_4 \cos\beta_2\right)\right] = M_{O_2} - M_{rf2} + l_2\left(\sin\beta_2 \sum_{j=3}^{4} F_{yj} - \right. \\ & \left. - \cos\beta_2 \sum_{j=3}^{4} F_{yj}\right), \\ & I_3\ddot{\beta}_3 + m_3 a_3\left(\ddot{x}_3 \sin\beta_3 - \ddot{y}_3 \cos\beta_3\right) + l_3 m_4\left(\ddot{x}_4 \sin\beta_3 - \ddot{y}_4 \cos\beta_3\right) = \\ & = M_{O_3} - M_{rf3} + l_3\left(\sin\beta_3 F_{y4} - \cos\beta_3 F_{y4}\right), \\ & I_4\ddot{\beta}_4 + m_4 a_4\left(\ddot{x}_4 \sin\beta_4 - \ddot{y}_4 \cos\beta_4\right) = M_{O_4} - M_{rf4}. \end{aligned}\right\} \tag{11}$$

The first two differential equations of the system (11) describe the motion of the aggregating tractor along the axes Ox and Oy, the third differential equation describes turning of the tractor around its mass centre. The three latter differential equations of the system (11) describe the turns of the component parts of the trailed hemp harvesting aggregate around its mass centre, namely: the hemp combine harvester itself, the hitch mechanism and the tractor trailer.

In such a way a system is produced from six differential equations (11) which describes the motion of the trailed hemp harvesting combine (machine), this aggregate having six degrees of freedom. These differential equations may be applied for other trailed tractor aggregates with a similar aggregation scheme as well (for instance, flax harvesting).

A numerical solution of this system of equations makes it possible to study regularities of the plane-parallel motion of each component in this machine aggregate in a horizontal plane. For this purpose we have developed a programme for numerical calculations and have made numerical calculations using a package of the applied programmes Maple 7 [11].

In addition, the following initial conditions were set:

when $t = t_0 = 0$:

$$x_1 = 0, \quad y_1 = 0, \quad \beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 = 0;$$

$$\dot{x}_1 = 0, \quad \dot{y}_1 = 0.$$

There were determined also the mass and design parameters of the hemp harvesting aggregate, as well as provisional resistance was determined by the results of the experimental research, which arises while pulling the hemp stalks, depending on the forward velocity of the hemp combine harvester.

After numerical calculations have been executed on the basis of real values of the parameters of the hemp harvesting machine KK-1.9, graphic dependencies were built of the parameters of a plane-parallel motion of the trailed hemp harvesting aggregate performing plane-parallel motion.

Figures 2 and 3 show dependencies from the time parameters: x_1, y_1, β_1, β_2, β_3, β_4 when $\alpha = \pi/4$.

It is evident from the graphic dependencies that at small angles α ($\alpha = \pi/40$) the turning angles of the aggregate component parts β_i $(i = \overline{1,4})$ also change smoothly in time by small values. The velocity of motion of the trailed hemp harvesting aggregate during this time interval becomes constant, too.

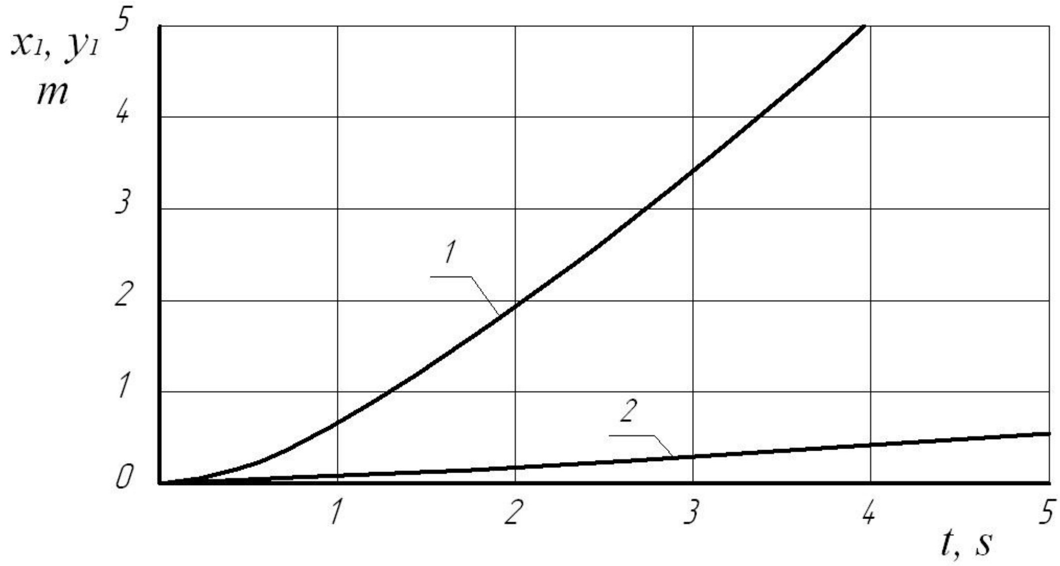

Fig. 2. **Dependence of the path covered by the mass centre of the aggregating tractor on time in the direction of axis x (curve 1), axis y (curve 2), at $\alpha = \pi/40$**

As it is evident from the presented graphs, at the beginning of motion of the hemp harvesting aggregate, when its starts moving from its place, the coordinate x_1 increases gradually, and only, when the aggregate has covered a path, equal to 0.5-0.7 m, increase of the coordinate x_1 becomes stabilised. At this moment the velocity of motion of the aggregate grows intensely, the coordinate y_1 remaining practically constant. After the aggregate has covered a path of 1.5-2.0 m, the velocity of its forward motion stabilises. The transitional process is completed after 2 seconds. All the turning angles of the aggregate components β_1, β_2, β_3, β_4 increase with time; however, the increase of the angles β_1 and β_2 is more intense.

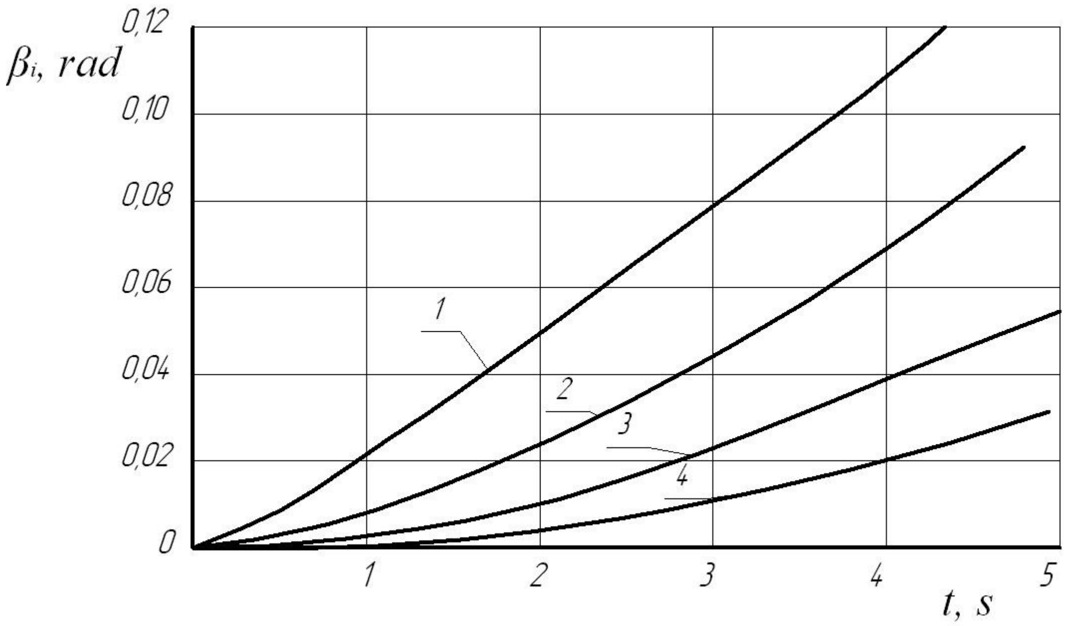

Fig. 3. **Dependence of the turning angles of the trailed hemp harvesting aggregate ($1 – \beta_1$, $2 – \beta_2$, $3 – \beta_3$, $4 – \beta_4$) on time at $\alpha = \pi/40$**

Thus, the obtained analytical dependencies and graphs of numerical calculations indicate that there are all technical possibilities for the choice of such design and kinematic parameters of the given hemp harvesting machine aggregate, which depending on the loading intensity of the pulling apparatus of the trailed hemp harvesting machine (which depends on the hemp yield), the values of the forces acting upon the wheels of this aggregate (which are characterised by their sizes, pressure in the tires, the soil bearing capacity, and its physical and mechanical characteristics during the harvest), the mass of the component parts of the aggregate, and the dimensions of its components, would ensure great rectilinearity of motion of the aggregate mass centres, stable change of the turning angles, which, on the whole, will ensure stable movement in a horizontal plane, reduced energy costs, and so on. All this will finally enable significant increase in the hemp harvesting quality: reduced losses of the mass of the plants and seeds, lower specific energy intensity (down to 10…15 %) at the expense of stable rectilinear motion, higher economic indicators of this valuable agricultural and industrial fibre crop.

Conclusions

1. In order to build a mathematical model of the plane-parallel motion of a trailed hemp harvesting aggregate, an equivalent scheme of the aggregate was built, its mass centres were set and its initial dimensions were determined. All the external stresses (forces and moments of forces) acting upon the wheels of the tractor as well as the wheels and operating tools of the trailed hemp combine harvester were determined in the accepted coordinate system. On the basis of the initial equations in the form of the Lagrange equations of the 2nd kind, generalised coordinates of the given dynamic system were set, expressions of the kinetic energy and generalised forces were determined, and after transformations a nonlinear system of differential equations of the second order was obtained.
2. As a result of the theoretical research an estimated mathematical model (a system of differential equations) of the motion of a trailed hemp harvesting aggregate is obtained which makes it possible at the stage of its designing to determine the regularities of the plane-parallel motion of each component part of this machine aggregate in a horizontal plane, and determine further the optimal design and kinematic parameters that ensure stability of the movement during the execution of the hemp harvesting technological process.
3. For numerical solutions of the obtained system of differential equations a programme for numerical calculations on the PC was created, and calculations were made according to which graphic dependencies of the generalised time coordinates were produced that make it possible to determine the initial moment of stable movement of the aggregate.

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