TUNEABLE ELASTOMERIC VIBRATION ISOLATOR

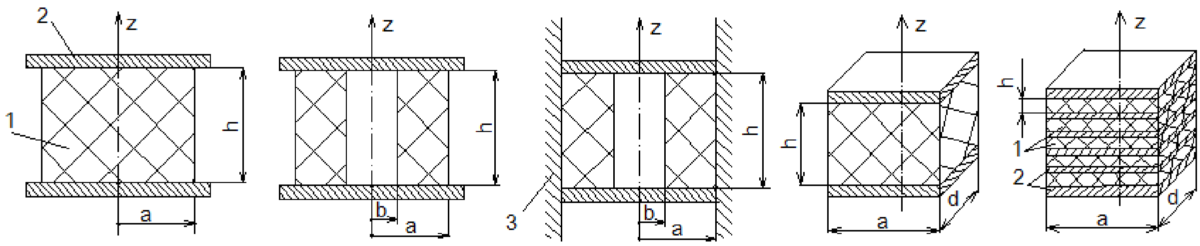
Vladimirs Gonca[1], Svetlana Polukoshko[2], Egons Lavendelis[1]
[1]Riga Technical University, Latvia; [2]Ventspils University College,
Engineering Research Institute "VSRC", Latvia
pol.svet@inbox.lv, vladimirs.gonca@rtu.lv

Abstract. Rubber and rubberlike materials (elastomers) are able to absorb input energy much better than other construction materials, therefore elastomers are widely used in manufacturing of the compensating devices, shock-absorbers, vibration isolators, joints, etc. In this paper the rubber – metal vibroisolators with movable nondeformable side surface, operating under axial compression, are considered. Such rubber-metal devices with adjustable stiffness allow to implement nonlinear characteristics "force-displacement" both of rigid and soft type even at small axial deformation, that cannot be obtained in classical forms of rubber-metal products. Based on the Ritz method, using the principle of minimum of total potential energy of deformation, the variational method of obtaining the stiffness characteristics of "force-displacement" is described. The weak compressibility of the rubber components is taking into account. As an example the solid cylindrical rubber - metal device with the movable outer side surface, working under axial compression, is presented. It is shown that under condition of axial compression the weak compressibility of the rubber layer neglecting may lead to significant errors in the calculation of "force-displacement" characteristics of the rubber-metal device.

Keywords: elastomers, vibration insulation, stiffness characteristics, sliding collar.

Introduction

Nowadays, under conditions of intensification of production processes, increasing of the equipment capacity and high-speed vehicle creation the question of preventing the harmful effects of vibration inevitably comes up. In most cases it is necessary to provide shock and vibration protection devices at the design stage. Vibration isolators containing flexible elements of rubber-like materials (elastomers) have become an integral part of almost all modern construction: machinery, machine tools, building structures and engineering constructions [1; 2]. Elastomers are the most suitable material for anti-vibration devices due to their adhesive characteristics, the ability to acquire a predetermined shape at press processing after curing, high elasticity and high internal damping [1-5]. The elastic properties of rubber characterized by a large difference between bulk and shear moduli (K/G ratio reach 500 ÷ 5000) [4; 5]. The high elasticity of rubbers allows them to withstand the large elongation without breaking; therefore rubber has greater capacity to store energy per unit volume compared with other materials. In this paper the vibration and shock absorber operating under axial compression is considered. Typical elements of such devices are cylindrical, prismatic, conical; some of them are shown in Fig. 1. The rubber cylinder or prism are reinforced by two perfectly rigid undeformable plates, which is cured on to the elastomers by vulcanizing.



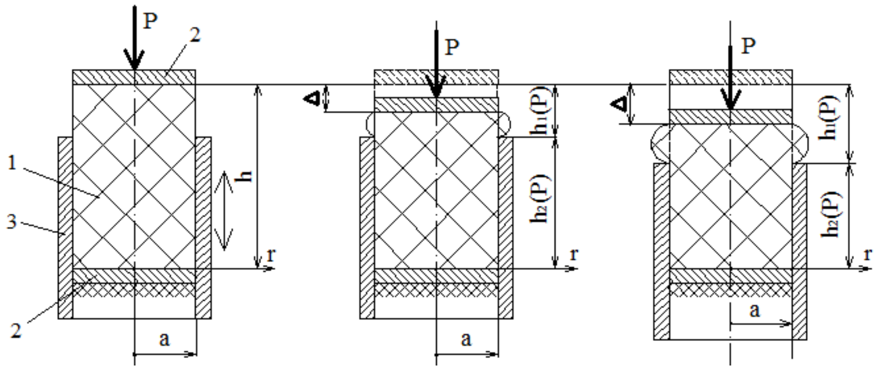
a) b) c) d) e)

Fig. 1. **Typical elements of reinforced rubber vibroisolators operating under axial compression:** a) solid cylindrical, b) hollow cylindrical, c) hollow cylindrical in a cage d) solid prismatic, e) thin-layered prismatic; 1 – elastomer; 2 – rigid plate; 3 – rigid fixed cage

Such devices are usually mounted between the vibrating base and protecting object. Natural frequency of vibrating mass depends on the stiffness characteristics of vibroisolator, i.e. analytical relationship between the imposed external compressive force and corresponding displacements. In many cases it is suitable to have the vibration and shock absorber with variable stiffness (tuneable), a variant of such absorber is discussed in [6; 7]. Improved approach to designing such devices is proposed in this work.

Materials and methods

In this paper we propose a design of a vibration absorber with rigid side boards, which can move parallel to the direction of the applied compressing force. Since due to axial motion of the rigid cage the free surface of the elastomeric layer changes, the stiffness of the absorber device changes (Fig. 2). In contrast to the shock absorbers with a fixed cage (fixed cage helps implement only the increase of stiffness in the process of loading), the moving cage makes it possible to increase or decrease stiffness (from initial stiffness) in the process of loading. Stiffness of the absorber can vary from hard (free lateral surface of the elastomeric layer of the absorber is zero, the layer deforms only due to the weak compressibility of the elastomer) to the stiffness of the shock absorber without a cage (height of the free elastomeric layer of the absorber is equal to the height of the elastomeric layer).



a) b) c)

Fig. 2. **Cylindrical vibration absorber with movable cage:** 1 – elastomer; 2 – upper and lower rigid plates; 3 –movable rigid ring cage

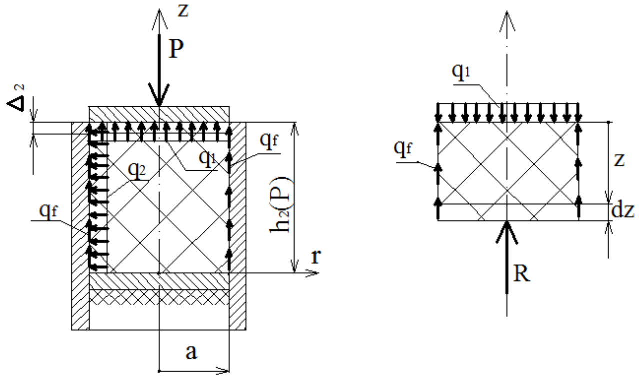


Fig. 3. **Scheme of forces acting on lower part of absorber**

To obtain the analytic dependence of "force-displacement" $P - \Delta$ for small finite strains (up to 10-15% in accordance with [4; 5]) the shock absorber is divided into two parts: upper part 1 of height h_1 with free surface of elastomer and lower part 2 of height h_2 with elastomer in the undeformable cage. In part 1 we have axisymmetric compression; in part 2 – volume compression. The solution was performed in [7], taking into account weak compressibility of the elastomer and neglecting friction forces on the contact surface of the elastomer cylinder and metal cage supposed perfectly rigid. Total vertical displacement of the elastomeric cylinder may be written as:

$$\Delta = \Delta_1 + \Delta_2, \tag{1}$$

where Δ_1 – displacement of the upper part with free side surface of elastomer;
Δ_2 – displacement of the lower part with elastomer in cage.

For the first part of the absorber the dependence $P - \Delta$ is found using the principle of minimum of total potential energy U, which consists of the potential energy of deformation J and external forces work. The absorber total potential energy is written as functional $U(u, w)$ of displacement functions u, w [4; 5]:

$$U(u,w) = J(u,w) - P\Delta_1, \tag{2}$$

$$J = 2\pi G \int_{-0.5h_1}^{0.5h_1} \int_0^a \left[\begin{array}{l} \left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)^2 + \\ + \frac{3\mu}{1+\mu} s \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}\right) - \frac{9\mu(1-2\mu)}{4(1+\mu)^2} s^2 \end{array} \right] r dr dz,$$

where G – elastomer shear modulus;
μ – Poisson's ratio;
s – hydrostatic pressure in elastomer.

Choosing the displacement functions *u* (*r*, *z*), *w* (*r*, *z*), respectively, along the axes *r* and *z* that satisfy the geometric boundary conditions, from the principle of minimizing the functional $U(u, w)$ we can find the dependence of the "force-displacement" $P - \Delta_1$:

$$\Delta_1 = \frac{Ph_1(P)}{\pi a^2 G} \left[1.8 + \frac{1.2 + 1.5\alpha^2}{\left(1 + 3\frac{1-2\mu}{2\mu}\alpha^2\right)} \right]^{-1}, \quad \alpha = \frac{a}{h_1(P)}. \tag{3}$$

For the second part of the shock absorber assuming that the side constraint is perfectly rigid, the dependence of the "force-displacement" $P - \Delta_2$ is defined as for the case of elastomer volumetric compression [2]:

$$\Delta_2 = \frac{3Ph_2(P)}{2\pi a^2 G} \frac{1-2\mu}{1+\mu}. \tag{4}$$

The dependences (3), (4) are obtained for small finite deformations, when:

$$0 \le \frac{\Delta_1}{h_1(P)} \le 0.10 \div 0.15. \tag{5}$$

Depending on the values of α and μ, contribution of the elastomer compressibility into (3) and (4) will be not less than the contribution of shear deformation. Therefore, neglect of the rest friction forces between the elastomer layer and the cylinder can lead to significant errors. Let us estimate this impact. For this purpose we consider the second part of the absorber depicted in Fig. 3, where q_1 – pressure from the elastomer on the cured rigid plate, q_2 – lateral pressure of the elastomer to the side surface of the cage, which presses the elastomer to the cylinder with the force P_s.

$$q_1 = \frac{P}{A_e}, \; A_e = \pi a^2, \tag{6}$$

$$P_s = q_2 A_s, \; A_s(P) = 2\pi a h_2(P),$$

where A_e – cross sectional area of elastomer;
A_s – lateral surface area.

On the contact surface of the cylinder and the elastomer contact statical friction force P_f appears:

$$P_f = fP_s = fq_2 A_s(P), \tag{7}$$

where f – coefficient of static friction of rubber on the material of the cylinder.

Let us estimate the effect of the friction force on the characteristic of "$P - \Delta$", assuming that the cylinder is perfectly rigid and the stress state is close to the volumetric compression of elastomer (Fig. 3).

Assuming that the problem is symmetric, we have:

$$\sigma_r\big|_{r=a} = q_2,\ \sigma_z = \frac{P}{A_e} = q_1,\ \varepsilon_r\big|_{r=a} = 0,\ \varepsilon_\theta = 0,$$

$$\varepsilon_r = \frac{2(1+\mu)}{G}\left(-\sigma_r + \mu(\sigma_\theta + \sigma_z)\right) = 0,\ \varepsilon_\theta = \frac{2(1+\mu)}{G}\left(-\sigma_\theta + \mu(\sigma_r + \sigma_z)\right) = 0. \quad (8)$$

From (6)-(8) we have: $\sigma_r = \frac{\mu}{1-\mu}\sigma_z$, or $q_2 = \frac{\mu}{1-\mu}q_1$, consequently:

$$P_f = fq_2 A_s = f\frac{\mu}{1-\mu}\frac{2Ph_2(P)}{a}. \quad (9)$$

The frictional force P_f obstructs the setting of the elastomeric layer in the cylinder, so it reduces the compression deformation of elastomer in the cylinder Δ_2, and consequently the total compression deformation of the vibroisolator Δ. From (6) and (10) we have:

$$\Delta_2^* = \frac{3(P-0.5P_f)h_2(P)}{2\pi a^2 G}\frac{1-2\mu}{1+\mu} = \frac{3Ph_2(P)}{2\pi a^2 G}\frac{1-2\mu}{1+\mu}\left(1 - f\frac{\mu}{1-\mu}\frac{h_2(P)}{a}\right). \quad (10)$$

Equation (10) allows simply enough to take into account the influence of the frictional force on the contact surface of the elastomer with the surface of another material, in which the elastomeric layer is located. For the considered construction of the damper neglect of frictional forces leads to errors in estimation of its vertical displacement and compressive stiffness.

It should be noted that dependence (9) and (10) may be used for small deformations (to 15%) and at pressure $q_1 \leq 20$ MPa, because if $q_1 > 20$ MPa the shear modulus G and the coefficient of friction f depend on q_1. The shear modulus may vary depending on the hydrostatic pressure s (in the case of axial compression $s=P/A_e$), the plot of such dependence is presented below [8]. In Fig. 4 for the elastomer of grade 2959 a plot of "$G(s)/G - s$" is given; here $G \approx 1.17$ MPa – shear modulus, calculated at low pressures. The first approximation of this dependence is linear and the shear modulus may be expressed as: $G(s) = G(1+\varphi_1 s)$, where $\varphi_1 \approx 2\cdot10^{-2}$ MPa^{-1}.

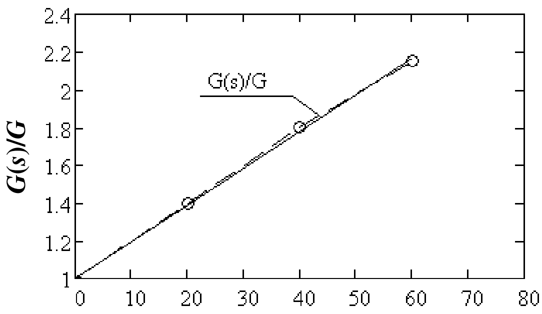


Fig. 4. **Plot of dependence $G(s)/G - s$:** ⊖○⊖ experimental points; — approximating curves

From the equations (3) and (10) we can derive the stiffness $C=P/\Delta$:

$$C = \frac{P}{\Delta_1 + \Delta^*_2},\ \text{let denote}\ C_1 = \frac{P}{\Delta_1}\ \text{and}\ C_2 = \frac{P}{\Delta^*_2},\ \text{then}\ C = \frac{C_1 C_2}{C_1 + C_2}, \quad (11)$$

$$C_1 = \frac{P}{\Delta_1} = \frac{G\pi a^2}{h_1(P)}\left(1.8 + \frac{1.2 + 1.5\left(\frac{a}{h_1(P)}\right)^2}{1 + 3\frac{1-2\mu}{2\mu}\left(\frac{a}{h_1(P)}\right)^2}\right),$$

$$C_2 = \frac{P}{\Delta^*_2} = \frac{2G\pi a^2}{3h_2(P)\dfrac{1-2\mu}{1+\mu}\left(1 - f\,\dfrac{\mu}{1-\mu}\dfrac{h_2(P)}{a}\right)}. \tag{12}$$

Without taking into account small compressibility of rubber stiffness this equation takes a form:

$$C^* = \frac{G\pi a^2}{h_1(P)}\left(3 + 1.5\left(\frac{a}{h_1(P)}\right)^2\right). \tag{13}$$

It is seen from (12) and (13) that stiffness of the absorber depends on the shear modulus G of elastomer, geometrical dimensions (parameter $\alpha = a/h_1$), Poisson's ratio μ. and the coefficient of static friction f . For filled rubbers Poisson's ratio ranges within $\mu \approx 0.465 \div 0.487$; for weakly filled rubber $\mu \approx 0.490 \div 0.4995$. The coefficient of statical friction f between the rubber slides and the steel surface with good lubrication ranges within ~ $0 \div 0.05$, without lubrication but with special treating of the rubber surface $\approx 0.2 \div 0.23$; without lubrication and special treating $\approx 0.35 \div 0.60$ [9].

Results and discussion

As an example the cylindrical rubber shock absorber with incrimental change stiffness is considered. Parameters of the absorber are: b = 18 mm, h = 40 mm, μ = 0.493, $G = 5.2 \cdot 10^5$ N·m^{-2}. The absorber is loaded with axial compressive force and the height of the side stop has changed "step by step", the step size is 6.5 mm. The experiment was performed on the test machine Zwick/Roell Z150 in the laboratory of the Riga Technical University. Then we compare the results obtained analytically with the results obtained during the experiment; the results are shown in Fig. 5. Analytical solution taking into account rubber low compressibility (without friction) coincides with the experimental data.

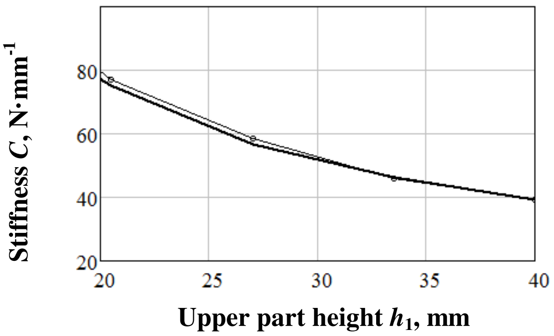


Fig. 5. **Stiffness dependence on force:** — experimental data, calculated data

Further for this absorber the influence of various factors (μ, f, a/h_1) on its stiffness is shown in the plots, built with help of Mathcad program.

In Fig. 6 the compressive stiffness dependence on the height of the upper free part of the damper is presented, calculated by three approaches: neglecting friction, with friction f = 0.05 and without taking into account weak compressibility. The difference between the three methods begins to appear when the height h_1 becomes less than ≈ 15 mm, i.e. $\alpha = a/h_1 > 1$. The plots in Fig. 7 show the contributions of the stiffness of the upper part (with free side surface) and lower part (with elastomer in the cage) into the total stiffness of the absorber; if $\alpha = a/h_1 < 1$ stiffness of part 1 becomes the main and stiffness of part 2 may be neglected.

In Fig. 8, a the compressive stiffness dependence on Poisson's ratio is shown for three heights of the free layer h_1 – 5 mm, 10 mm and 15 mm (i.e. $\alpha = a/h_1$ = 3.6; 1.8; 1.2), the friction coefficient for all cases is f = 0.2. The Poisson's ratio value influence on stiffness increases when h_1 decreases (α increases, $\alpha > 1$). In Fig. 8b the compressive stiffness dependence on the coefficient of friction is shown for three heights of the free layer: 5 mm, 10 mm and 15 mm; for all cases μ = 0.487. Static friction coefficient f value influence on stiffness also appears when α >1. The thinner the layer, the greater the influence of both the Poisson's ratio μ value and the coefficient of friction f value.

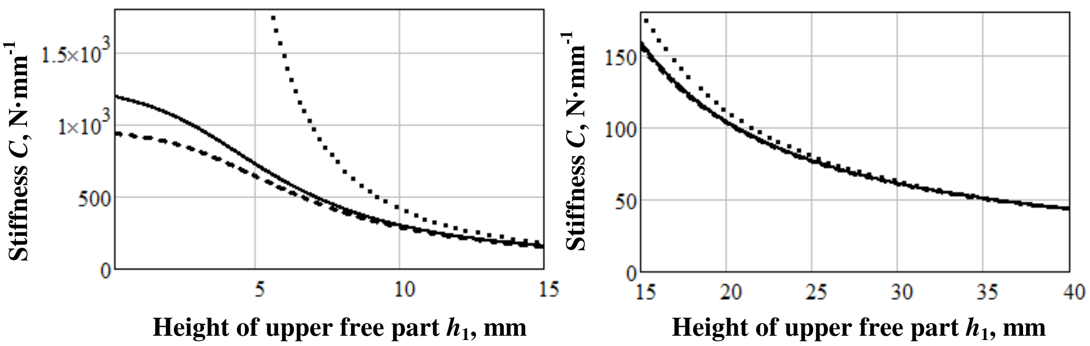


Fig. 6. **Compressive stiffness dependence on height of upper free part of damper:** —— with friction $f = 0.05$; - - - neglecting friction; ···· without weak compressibility

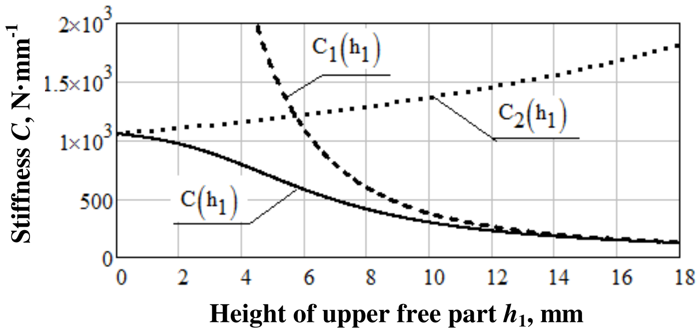
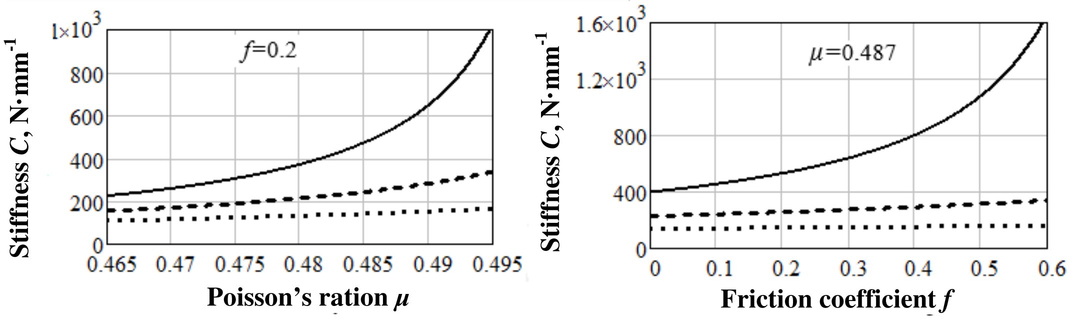


Fig. 7. **Contribution of upper and lower parts to total compressive stiffness:** —— common stiffness $C(h_1)$; - - - stiffness of part 1 $C_1(h_1)$; ···· stiffness of part 2 $C_2(h_1)$



a) b)

Fig. 8. **Compressive stiffness dependence: a) on Poisson's ratio μ at $f = 0.2$; b) on friction coefficient f at $\mu = 0.487$ for:** —— $h_1 = 5$ mm; - - - $h_1 = 10$ mm; ···· $h_1 = 15$ mm

The given above graphs illustrate the relationshs (11), (12) and (13), which are obtained based on "force-displacement" dependences $P - \Delta$ of the equations (3), (4), (10). Experimental verification was performed for the insulator with weakly-filled rubber with $\mu = 0.493$. The results received confirm this theoretical dependence.

Conclusions

In this paper a tuneable vibration absorber with adjustable compression stiffness is discussed. Perfectly rigid moving (parallel to the vertical axis z) vertical side stops (rigid ring for the rubber cylinder) allow to change the stiffness characteristic "force-displacement".

A method for determination of this dependence is proposed, taking into account low compressibility of the material of rubber layers and also friction forces between the elastomer and ring. The received solutions may be used in designing of vibration absorbers with a desired non-linear ("hard" or "soft") stiffness.

Neglecting the weak compressibility of rubber leads to a large error when the thickness of rubber decreases. Calculation of stiffness taking into account the friction force is important for damping of the impact dynamic processes and when the elastomeric part of the damper is performed from highly filled rubber.

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