DOUBLE PENDULUM VIBRATION MOTION IN FLUID FLOW

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Abstract. The paper analyses the double pendulum motion in the vertical plane interacting with the fluid (air or water) flow. The pendulum consists of two plates, fastened together with each other and with the foundation by the axes. The case was investigated where the fluid flows in a horizontal direction but the plates move around the pivots in the vertical plane. It is assumed that there is a laminar fluid flow. Double pendulum interaction with the flow is described as localized particles interaction with the plates, using the mass centre and angular momentum exchange theorems of mechanical systems. A system with two degrees of freedom is investigated. Interaction with fluid is approximated by a square step dependence on the flow relative velocity taking into account the direction of motion. The resulting system of equations is used to simulate the motion. The three cases of motion are analysed: free oscillations against a stationary fluid to determine experimentally all physical parameters of the interaction; auto vibration motion at a constant flow velocity; time variable flow induced oscillations. Some interesting results of the plate motion graphics are given.

Keywords: double pendulum, plate interaction with fluid, flow-induced vibrations.

Introduction

Object vibrations in surrounding environment include motion inside fluid at low speed region with a range when moving parts of elements change the velocity direction. Therefore, it is necessary to analyze the mechanical system motion as a double pendulum (1) vibration in fluid flow (2) (Fig. 1). Nature observation shows that some of the leaves of the trees start pendulum motion (rattle) in windy weather. We can also observe water plant vibrations in the river by water stream flows. It is important to check up such vibrations for untraditional energy production (without propeller system) [1]. For that it is necessary investigating some simpler systems like a double pendulum motion using the classical mechanics theory [2-3]. This report deals with analysis of two plates fastened together with each other and with the foundation connected to the pivots (that is, with the axis perpendicular to the direction of movement). The double pendulum and fluid interaction forces are analyzed and their reduction to the pendulum pivot points is found out. After that, considering wind interaction forces differential equations for the double pendulum motion analysis are given. Some results of modelling are shown.

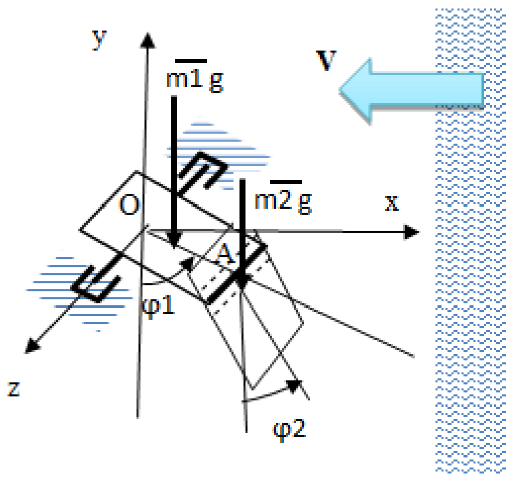

Fig. 1. **Double pendulum in fluid flow:** 1 – double pendulum; 2 – horizontally flow

Model of object

The object consists of two rectangular plates 1 and 2 (Fig. 2.). The plates are rotated around the horizontal axes Oz and Az. As generalized coordinates angles $\varphi1$ and $\varphi2$ are used. In the plates for the mass centers $C1$ and $C2$ vertically gravity forces $m1g$ and $m2g$ are applied respectively. The rotary axes Oz and Az can be placed asymmetrically with distances Δ1 and Δ2 from the plates ends. Fluid flow is horizontal with a speed V. The analysis of the motion of the double pendulum system by differential equations requires finding the fluid flow interaction forces. They are obtained in the next section.

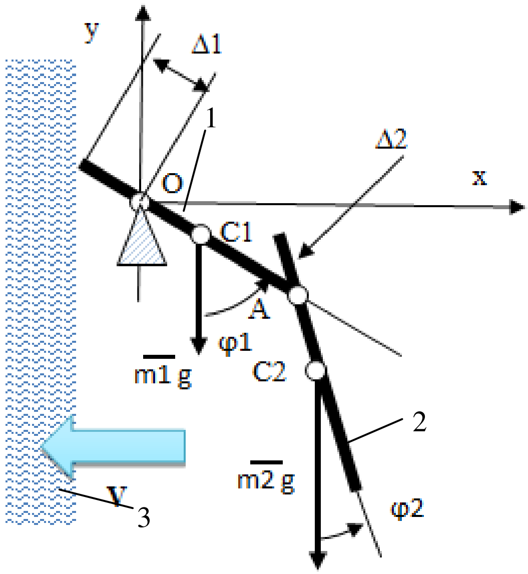

Fig. 2. **Plane model of pendulum:** 1 – pendulum rotating part; 2 – pendulum plane motion part; 3 – horizontal flow; $\varphi 1$, $\varphi 2$ – generalized coordinates

Reduction of fluid flow interaction forces

Fluid flow interaction forces depend on the relative speed of the contacts points. The main interaction is the square function of transverse speed components, perpendicular to the plate. Therefore, it is necessary to find out the point velocities projections on the perpendicular directions AO and AB (Fig. 3.). It means that for the plate AO expression from two graphs 1 and 3 (Fig. 3) can be found. Accordingly, for the plate AB all three graphs 1, 2 and 3 can be used. In this double pendulum task for forces reduction it is convenient to use the points O and A as reduction centers. Then, the fluid flow principal vectors *RV*1, *RV*2 and the principal moments *MV*1, *MV*2 can be found from the following integrals (for Fig. 3):

$$\frac{-RV1}{C1} = \int_0^{L1} (\omega 1 \cdot \xi 1 - V \cdot \cos(\varphi 1))^2 d\xi 1$$

$$\frac{-MV1}{C1} = \int_0^{L1} (\omega 1 \cdot \xi 1 - V \cdot \cos(\varphi 1))^2 \cdot \xi 1 d\xi 1$$

$$\text{Find}(RV1, MV1) \rightarrow \begin{pmatrix} C1 \cdot L1^2 \cdot V \cdot \omega 1 \cdot \cos(\varphi 1) - \dfrac{C1 \cdot L1^3 \cdot \omega 1^2}{3} - C1 \cdot L1 \cdot V^2 \cdot \cos(\varphi 1)^2 \\ \dfrac{2 \cdot C1 \cdot L1^3 \cdot V \cdot \omega 1 \cdot \cos(\varphi 1)}{3} - \dfrac{C1 \cdot L1^4 \cdot \omega 1^2}{4} - \dfrac{C1 \cdot L1^2 \cdot V^2 \cdot \cos(\varphi 1)^2}{2} \end{pmatrix}; \quad (1)$$

$$\frac{-RV2}{C2} = \int_0^{L2} (\omega 2 \cdot \xi 2 + f20)^2 d\xi 2$$

$$\frac{-MV2}{C2} = \int_0^{L1} (\omega 2 \cdot \xi 2 + f20)^2 \cdot \xi 2 d\xi 2$$

$$\text{Find}(RV2, MV2) \rightarrow \begin{pmatrix} \dfrac{C2 \cdot L2^3 \cdot \omega 2^2}{3} - C2 \cdot L2^2 \cdot f20 \cdot \omega 2 - C2 \cdot L2 \cdot f20^2 \\ \dfrac{C2 \cdot L2^4 \cdot \omega 2^2}{4} - \dfrac{2 \cdot C2 \cdot L2^3 \cdot f20 \cdot \omega 2}{3} - \dfrac{C2 \cdot L2^2 \cdot f20^2}{2} \end{pmatrix}. \quad (2)$$

$$f20 = \omega 1 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - V \cdot \cos(\varphi 2)$$

Here

$$\omega 1 = \dot{\varphi}1; \quad \omega 2 = \dot{\varphi}2; \quad f20 = \omega 1 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - V \cdot \cos(\varphi 2),$$

where $L1$, $L2$ – length of plates;
$\omega 1$, $\omega 2$ – angular velocities;
$C1$, $C2$ – constants, including drag coefficients, density of fluid;
ξ – width of plates.

If the rotation axis of the plates are not at the ends, the flow principal vectors $RV1$, $RV2$ and the principal moments $MV1$, $MV2$ can be found by separated length (-Δ1 till 0, and -Δ2 till 0) from formulas (1) and (2), taking into account deviations Δ1 and Δ2 (see Fig. 2.).

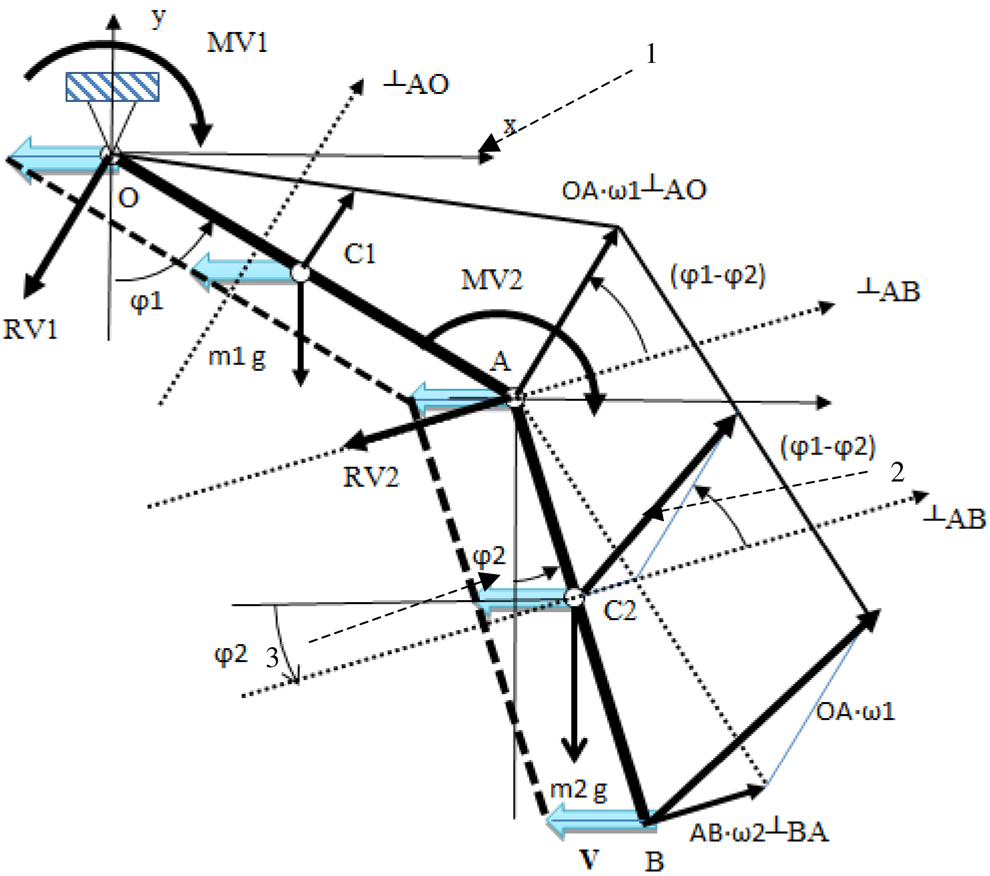

Fig. 3. **Graphics of double pendulum points velocities components:** 1 – graphic of first pendulum rotation velocity component; 2 – second pendulum relative rotation velocity component; 3 – horizontal flow component; $RV1$, $RV2$ – principal vectors; $MV1$, $MV2$ – principal moments

Differential equations of pendulum motion

Differential equations of motion can be found, for example, by the D'Alembert's principle of the given mechanical system (see Fig. 4.).

Sum of forces moments against the point A for only the second plate 2 gives:

$$\begin{gathered} -MV2 - \left(JC2 + m2 \cdot r2^2\right) \cdot \ddot{\varphi}2 + \\ + \phi AOn \cdot r2 \cdot \sin(\varphi 1 - \varphi 2) - m2 \cdot \ddot{\varphi}1 \cdot r1 \cdot \cos(\varphi 1 - \varphi 2) - m2g \cdot r2 \cdot \sin(\varphi 2) = 0. \end{gathered} \tag{3}$$

Sum of forces moment against the point O for both plates gives:

$$\begin{aligned}
&-MV1-MV2-RV2\cdot L1\cdot\cos(\varphi1-\varphi2)-\left(JC1+m1\cdot r1^2\right)\cdot\ddot{\varphi}1+\\
&-\left(JC2+m2\cdot r2^2+m2\cdot r2\cdot L1\cdot\cos(\varphi1-\varphi2)\right)\cdot\ddot{\varphi}2+\\
&-\phi C2An\cdot L1\cdot\sin(\varphi1-\varphi2)-m2\cdot\left(L1\cdot r2\cdot\cos(\varphi1-\varphi2)+L1^2\right)\cdot\ddot{\varphi}1+\\
&+\phi AOn\cdot r2\cdot\sin(\varphi1-\varphi2)+\\
&+\phi AOn\cdot L1\cdot\cos(\varphi1-\varphi2)-m1g\cdot r1\cdot\sin(\varphi1)+\\
&-m2g\cdot(r1\cdot\sin(\varphi1)+r2\cdot\sin(\varphi2))=0.
\end{aligned} \tag{4}$$

Here (see Fig. 4.):

$$M\phi C1=JC1\cdot\ddot{\varphi}1;\quad \phi C1Ot=m1\cdot r1\cdot\ddot{\varphi}1;\quad \phi C1On=m1\cdot r1\cdot\dot{\varphi}1^2;$$

$$M\phi C2=JC2\cdot\ddot{\varphi}2;\quad \phi C2At=m2\cdot r2\cdot\ddot{\varphi}2;\quad \phi C2An=m2\cdot r2\cdot\dot{\varphi}2^2;$$

$$\phi AOn=m2\cdot L1\cdot\dot{\varphi}1^2;\quad \phi AOt=m2\cdot L1\cdot\ddot{\varphi}1,$$

where $JC1$, $JC2$ – central moments of inertia mass m1, m2;
$r1$, $r2$ – distance from center masses till pins;
$L1$ – length of first plate;
$\ddot{\varphi}1$, $\ddot{\varphi}2$ – angular accelerations of plates.

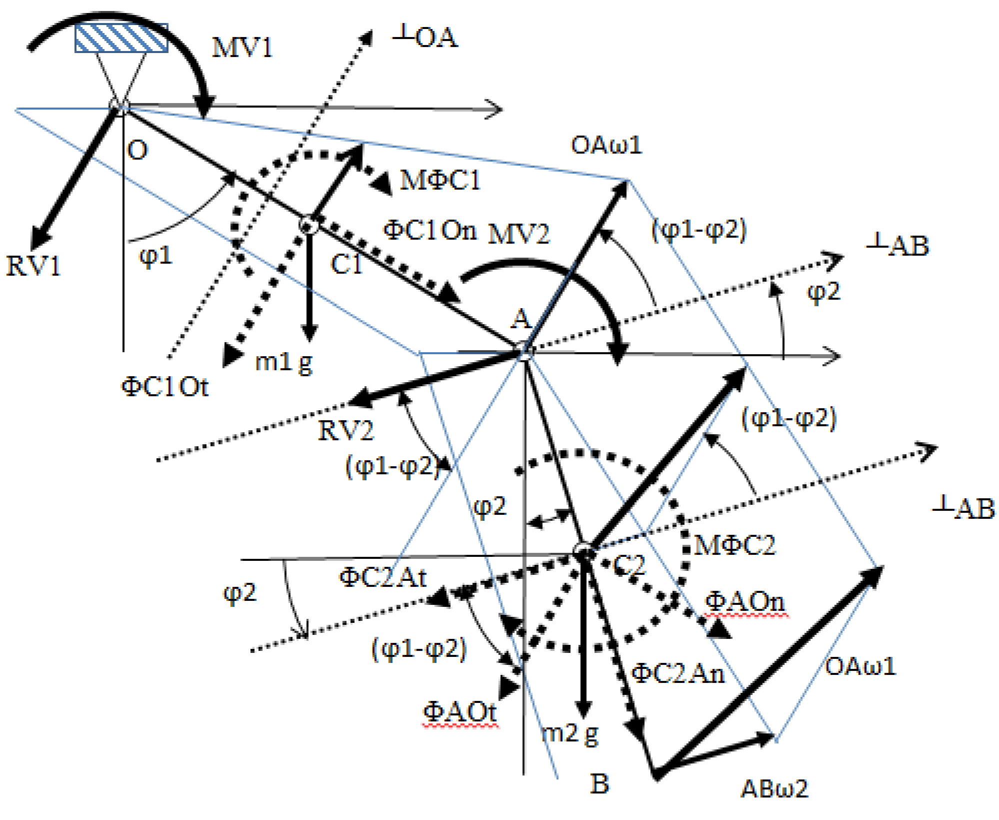

Fig. 4. **D'Alembert's principle forces:** ($m1g$, $m2g$, $RV1$, $RV2$, $MV1$, $MV2$) – active forces and their moments; ϕ, $M\phi$ – components of forces of inertia and their moments

From two equations (3) and (4) the angular accelerations $\ddot{\varphi}1$, $\ddot{\varphi}2$ can be found. Then, by integrations $\ddot{\varphi}1$, $\ddot{\varphi}2$ the motion parameters can be calculated, using numerical computer calculation methods (for example, Euler method). There exists an additional possibility of analysis of this motion by special fluid flow analysis computer programs, for example, "Working Model".

Motion modelling

In this research the investigation was made by solid bodies plane motion calculation computer program “Working Model 2D”.

Some interesting graphics of motion are shown in Fig. 5-7. Comments are given below for each figure.

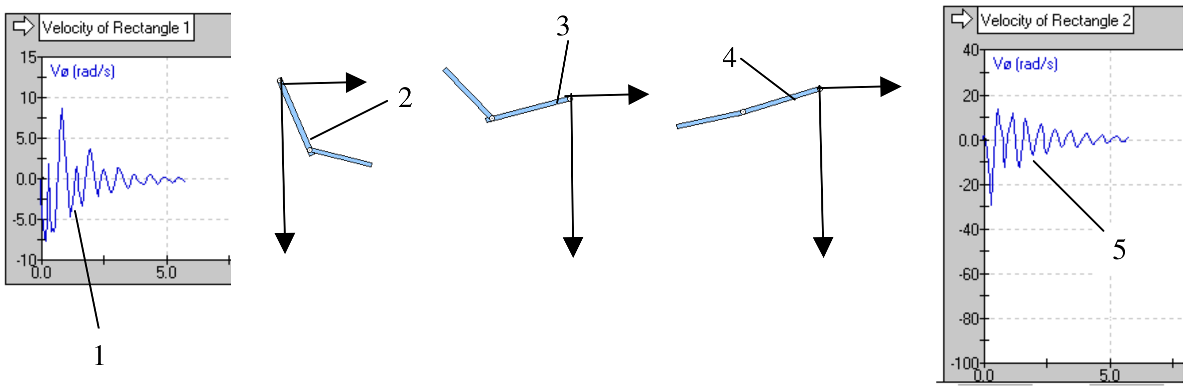

Fig. 5. **Transitional damped vibration motion of pendulum:** 1 – angular velocity of first plate 1 as time function; 2 – initial position of double pendulum; 3 – middle position of pendulum; 4 – close end rest position of pendulum; 5 – angular velocity of second plate 2 (see Fig. 2)

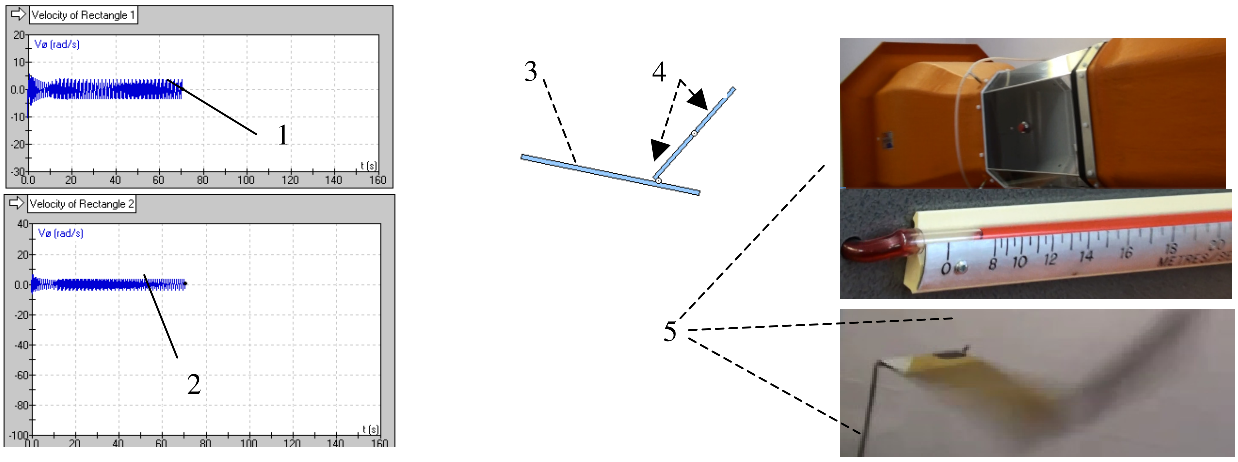

Fig. 6. **Stationary vibration motion of pendulum with large amplitude:** 1 – angular velocity of first plate 1 as time function; 2 – angular velocity of second plate 2 as time function; 3 – initial position of double pendulum; 4 – positions of rotation axis are with eccentricities Δ1 and Δ2 (see Fig. 2); 5 – experiment in wind tunnel with flow velocity about 6 m·sec^{-1}

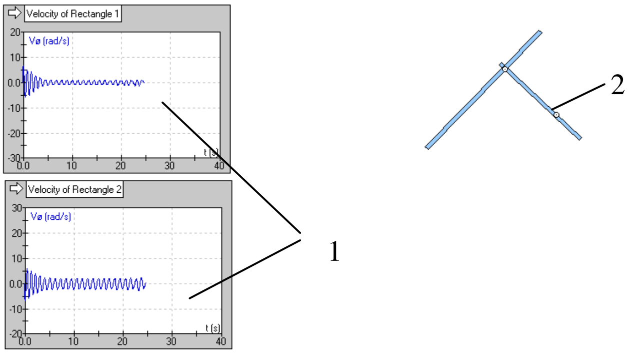

Fig. 7. **Bifurcation of vibration motion (with small amplitude):** 1 – angular velocities; 2 – initial position of double pendulum

Results and discussion

Analysis of the modeling results shows that it is possible to obtain a stationary oscillations motion in the double pendulum system. For that the axis of rotation can be displaced by eccentricity Δ1, Δ2 against the end points of the plates. This means that fluid flows at a constant speed in the observed system generating periodically vibration motion. In this system the existence of vibration bifurcations should be checked out. This question requires additional investigations and is not covered by the given article.

Conclusions

1. Double pendulum vibration system by using fluid flow interaction can be used for untraditional energy production (without propeller system).
2. Stationary vibration motion depends on special pendulum rotation axis displacements.
3. Theory validation was checked by the wind tunnel.

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