MATHEMATICAL MODEL FOR SOLAR ENERGY COLLECTOR WITH REFLECTORS

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Abstract. In order to increase the efficiency of a flat plate solar collector, it is recommended to equip it with two mirrors so that the reflected from the mirrors solar radiation strikes the rear surface of the collector. There theoretical motivation of the development of a collector with reflectors is worked out, where in order to increase the intensity of solar radiation on the collector's absorber, reflectors are used. In this case the collector has to be equipped with the system for tracking the sun. In order to calculate the main parameters of the solar collector with reflectors and compute the amount of the produced heat energy by the collector, its mathematical model is worked out.

Keywords: solar radiation, solar collector, efficiency, mathematical model.

Introduction

Today's solar collector devices are mainly used for the production of hot domestic water, space heating using heated floors, for swimming pool water heating, as well as drying and cooling. In Latvia the sun radiation has relatively low intensity. The total energy of the sun is about 1109 $kWh \cdot m^{-2}$ per year that is slightly more than in the Scandinavian countries. For the use of solar thermal energy the period is from the last decade of April, when the radiation intensity is around 120 $kWh \cdot m^{-2}$, till the first ten days of September. During this period, about 1800 hours of solar energy can be used for production of hot water by installing solar panels [1]. The cost of the existing technologies decreases gradually, as they are replaced by new and more completed systems. The paper research direction is focused to solar energy conversion into thermal energy for water heating, as well as for a variety of other business purposes. The planned solar installation share of the load in relation to the additional heating system is a variable depending on the season. The addition part of the heat source load is usually covered by the operation of the gas, wood or liquid fuel boiler or electricity.

The goal of our investigation was to look for possibilities to increase the efficiency of a flat plate solar collector by the intensification of radiation falling on the collector's absorber.

Materials and methods

One of the research tasks is to create an effective solar water heating collector with increased efficiency. As the efficiency of a collector in large scale depends on the intensity of solar radiation striking the absorber, it was decided to increase the radiation intensity on the absorber by the use of reflectors (mirrors). In this case reflected from the reflectors solar radiation is striking the absorber of the collector from its rear side, too. Then there is no need for heat insulation of the rear side of the collector, because there is the so called box-type absorber used from both sides covered by transparent glass covers. From the front side the absorber receives direct solar radiation, but from the rear side – reflected. The principal scheme of the equipment is shown in Figure 1. The collector is supplied with a tracking the sun mechanism.

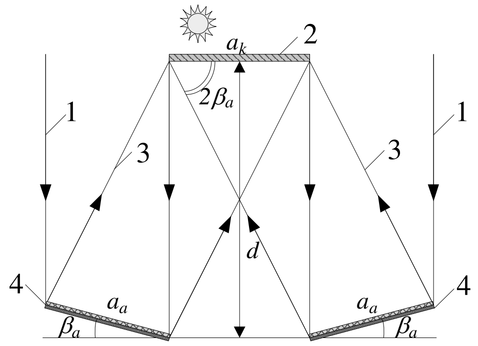

Fig. 1. **Principal scheme of solar collector with reflectors:** 1 – falling sun rays; 2 – box type absorber; 3 –reflected sun rays; 4 –reflectors

The angles of rotation of the collector are calculated:

$$\beta_{a1} = \arcsin \frac{a}{2d}, \tag{1}$$

and

$$\beta_{a2} = \frac{1}{2} \operatorname{arctg} \frac{a}{d} \tag{2}$$

From formulae (1) and (2) it follows that the right and left reflector placement angle values depend on the width of the collector absorber *a* and placement distance *d*. The values *a* and *d* are conditional width and length of the absorber and reflector. If both are the same, the rotation angle for the right reflector β_{a2} = 22.5º and the left reflector β_a1 = 30º. As it is seen, the right and left side angles of rotation of the reflectors are not equal. This can be explained by the fact that β_{a1} – is maximal possible angle of rotation and β_{a2} – is minimal possible angle of rotation. When both reflectors of the solar collector irradiated from both sides turn at the same angle (see Fig. 1), then denoting the width of the collector absorber by a_k, and the width of the reflectors with $a_{a1} = a_{a2} = a_a$., as well as the rotation angle of the reflectors – with β_a, and using geometric correlations, the ratio between the width of the reflector and absorber are as follows:

$$\frac{a_a}{a_k} = \frac{\cos 2\beta_a}{\cos \beta_a}. \tag{3}$$

An important condition for the equation (3) is that the length of the reflecting surface is never bigger than the width of the absorber, that is $a_a{\cdot}a_k^{-1} \leq 1$. This insures the most efficient use of the reflective surface area.

In the work the ratio of concentration is expressed. For determination of this, the concentrators of reflector reflectance – ρ_{at} is taken into account, which depends on the spectral reflectance of the reflectors [2]. The results are summarized in the common graph (Fig.2), which shows the ratio of the concentration depending on the number of reflectors, for two and four reflectors accordingly. In the calculations it is assumed that $\rho_{at} = 0.9$. Dependence of the distance *d* on the rotation angle of the reflectors is shown in Figure 2. It follows that in this case the optimal turning angle of the reflector $\beta_a \approx$ 10-17º, providing the concentration ratio from 1.5 to 3.

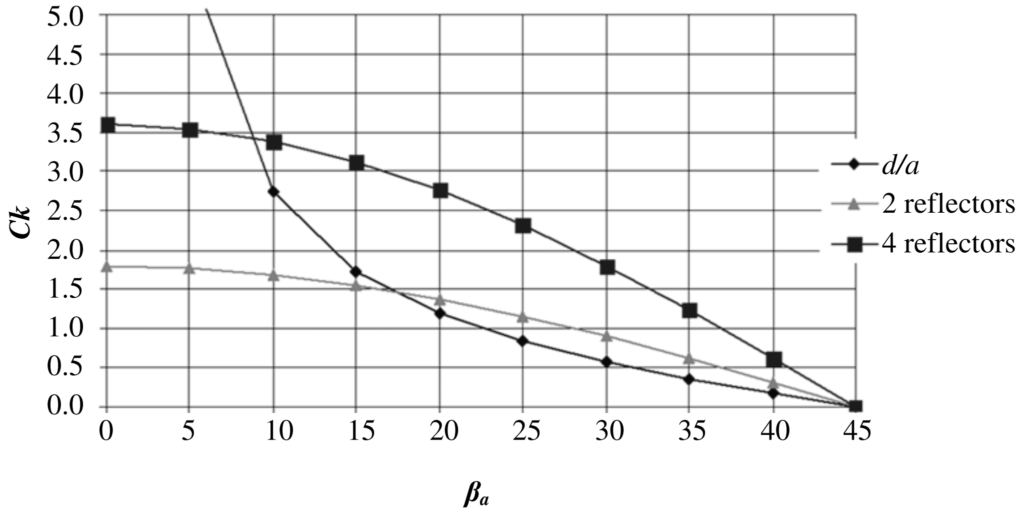

Fig. 2. **Characteristics of the solar collector with reflectors:** *Ck* – concentration ratio of reflectors; β_a – rotation angle of the reflectors; *d/a* – ratio between the distance of the reflector and the width of the solar collector

On the basis of studies and engineering calculations, a patent of the Latvian Republic LV 13711B is received for invention of the solar collector with a two-part absorber [3]. The absorber of the collector is made up from two parts, one to the other positioned at some angle to the working surface and focused on the reflector side. The angle between the absorber parts is selected so that the reflected radiation from absorbers to the flat surface falls below the smallest possible angle. For example, if from each side in a row there are three reflectors placed, the angle is selected so that the reflected sun rays from the second reflector are perpendicular to the front surface of the absorber. In the case of reflecting the elements are placed on the same parabolic-shape base, thus creating a parabolic surface much higher ratio of concentration can be obtained [2; 4].

Thermal analysis is based on the solar collector parameters change depending on the angle of incidence. Reflection ρ_p, absorption α_p and transmittance τ_p coefficients of a solar collector glass cover are calculated. The calculations show that the transmission coefficient of the glass will decrease, when the beam angle to the solar collector will be bigger than 50º.The reflection coefficient at this angle will grow rapidly. But as the solar collector will be provided by a tracking system, then, as it can be seen from the graph (Fig.3), the changes are insignificant – up to 10º. In order to specify the glass, which will be used in a particular installation, the transmission coefficient of the solar radiation calculations were made according to the angle of incidence of the solar beams with a step 0.5º, and the averaged values were stated.

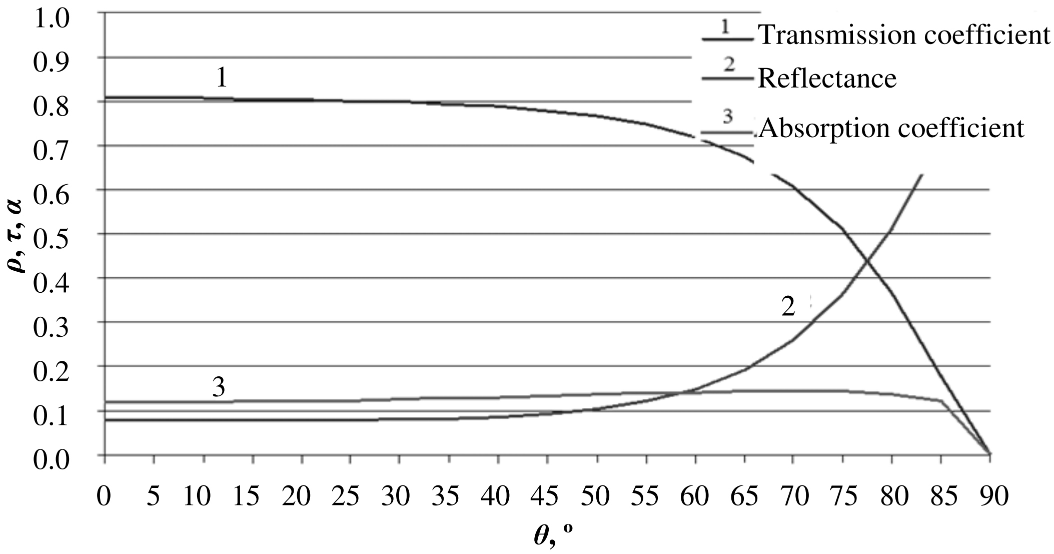

Fig. 3. **Results of the calculations of reflectance ρ_p, absorption α_p and transmission τ_p coefficients**

The heat energy absorbed by the absorber of the solar collector was analyzed. The biggest part of the solar radiation striking the working surface of a collector is absorbed into the absorber material and further transmitted to the heat carrier, which is known as the useful energy. In thermal installation heat losses have to be taken into account. They depend on the individual design of the installation, the coefficient of heat transfer between the components of the installation, the calculated heat loss coefficient values at different outdoor temperature and wind speed.

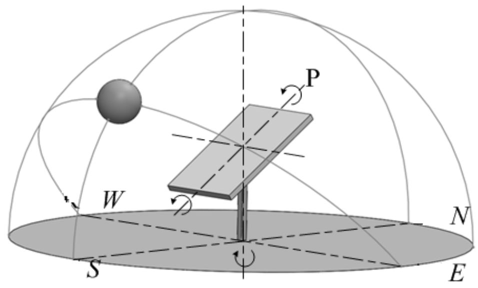

Fig. 4. **Principal scheme for full tracking system**

In order a solar collector produces as much as possible heat energy during a day, it must be oriented so that the sun rays are striking the working surface of the collector perpendicularly. Then similar to the other factors, the efficiency of the equipment will be at its maximum. The receiving systems are categorized by the type of tracking – one axis (single-axis) and two (two-axes) as shown in Figure 4. Single-axis tracking systems are used at horizontal position, considered in the thesis – the east-west (the *E-W horizontal*), south-north (the *S-N horizontal*), and parallel to the Earth's axis or polar axis (the *E-W polar*) [5; 6].

In order to evaluate the amount of heat energy received by following the sun surface, in all the cases a variety of correlations are used. The results of calculation for tracking the sun systems are given in Table 1. The two-axes tracking system collects the maximum possible solar energy during the day at summer solstice and equinox. The *E-W polar* tracking system makes it possible to collect maximum solar energy during the equinox, which is equal to the two-axes tracking system collected, and during the summer solstice by about 8 % less radiation energy compared with the two-axes tracking system. This is explained by the fact that inclination of the solar energy collector is equal to the polar axis, which in this case is equal to the place latitude, and on equinox, when $\delta = 0°$ and the angle of solar incidence $\theta = 0°$.

Table 1

Comparison of solar energy received by various modes of tracking

Tracking mode	**Percentage comparison to full tracking system**	
	SS	E
Full tracking	100	100
E-W polar	92	100
N-S horizontal	74	82
E-W horizontal	91	68

SS – summer solstice; E – equinoxes

When analyzing the results which are summarized preference would be given to *E-W polar* single-axis tracking system, because the obtained results in percentage are closer to the two-axes tracking system compared with other single-axis tracking systems.

Results and discussion

In order to substantiate the calculation of the efficiency of a solar collector, the maximum of heat energy, which the equipment is able to produce, was calculated. Assuming that the rotation mechanism of the solar collector 4 (Fig.5.) will operate with interruptions, the angle of solar beam incidence has been analyzed. In the case when the absorber of a solar collector is perpendicular to the sun rays and $\theta = 0°$, solar radiation that strikes the reflective surface as I_k (the left side reflector) and I_l (the right side reflector), comes to points O_1 and O_2. Being reflected they come to the solar collector as the values I_{ka} and I_{la} in point O. The sun beam after some time gets in the second position, when I'_k becomes like I'_{ka} and I'_l like I'_{la} in point O'. The solar incidence angle will change and in Figure 5 it is displayed as angle θ.

Taking into account the correlations that determine the intensity of solar radiation which reaches the collector working surface, as well as the used thermodynamic parameters and the factors influencing them, the mathematical model for determination of the amount of heat energy produced by the solar collector with reflectors is created as follows:

$$Q_k = 3.6 \cdot S_k \left[\begin{array}{c} \frac{24}{\pi} I_{SC} \left(1 + 0.033 \cos\left(\frac{360 \cdot n}{365} \right) \right) \cdot \\ \cdot \cos\theta \tau_A^{m_A} (1.01 \cdot \tau_{p1} \alpha_a + 1.01 \cdot \tau_{p2} \alpha_a 2\rho_{at} \cos 2\beta_a \cos 2\theta) - \\ - q_z (T_i - T_g) \end{array} \right], \quad (4)$$

where Q_k – amount of heat accumulated by the solar energy collector, J;
S_k – working surface area of the solar energy collector, m^2;
τ_A – atmospheric transmission coefficient;
θ – angle of solar incidence, degrees;
I_{SC} – solar constant, W·m^{-2};
n – serial number of the day, counting from the first January;

τ_p – transmission coefficient of the solar collector covering;
q_z – energy loss coefficient, W·m^{-2}·K^{-1};
T_g – outdoor air temperature, K;
T_i – inlet temperature of the heat carrier into the collector, K;
m_A – atmospheric mass, relat.unit, the value of which is determined according to the day of the year (in view of the sunset and sunrise times, or angles ω_{sr}):

$$m_A = \frac{1}{\frac{\omega_{sr}\pi}{180}\sin\varphi\sin\delta + \cos\varphi\cos\delta\cos\omega_{sr}}, \tag{5}$$

where ω_{sr} – sunset hour angle, degrees;
φ – latitude, degrees;
δ – solar declination, degrees or radians.

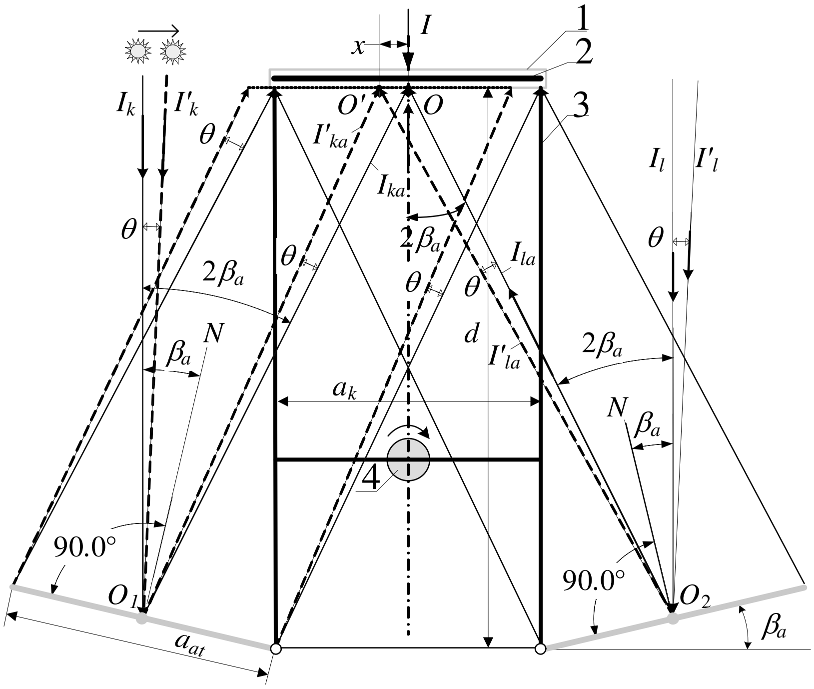

Fig. 5. **Principal scheme of operations for the solar collector with reflectors:** 1 – solar energy collector irradiated from both sides; 2 – absorber; 3 – frame; 4 – rotation mechanism

For determination of the theoretical amount of heat that can be obtained from a solar collector with reflectors, the simulation model was used. The model was established with Matlab-Simulink software, using the equation (4). In order to simplify the simulation process, the simulation model was divided into three blocks, which are identified by letters A, B and C.

Figure 6. shows the block A, block B and block C combination, thus demonstrating the theoretical heat quantification of the solar collector with reflectors. The simulation model is developed in accordance with formula (4). The simulation model block A (see Fig.6.) assesses the ratio of concentration, considering the dependence of the rotation angle of the reflector on the other parameters of the solar collector. On the basis of this unit determined correlations β_a and C_k are used. Incepting parameters like latitude and number of the day, with assistance of the simulation block B the amount of solar radiation, collected by the solar collector with reflectors during a day is determined. The simulation block C contains the heat loss coefficient determination scheme, where correlations are found for calculation of the heat loss coefficient of the solar collector shell, the heat loss from the collector absorber surface coating and the factors influencing the heat losses (wind, temperature).

From the data obtained from the simulation model processing results it follows that the most productive months are from March to October.

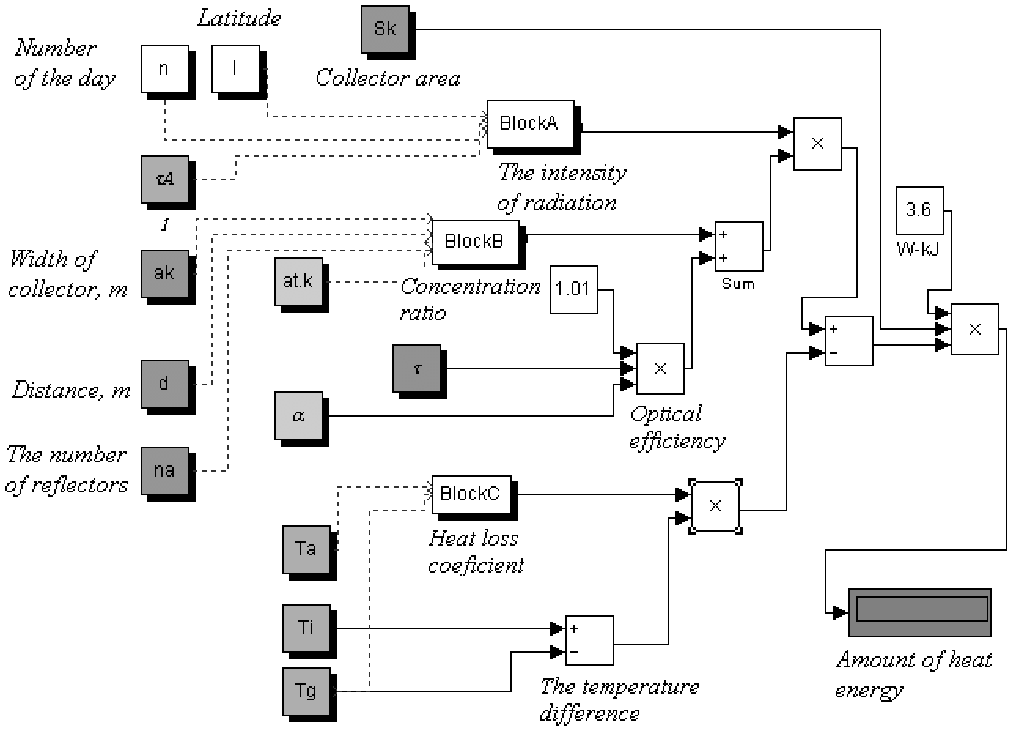

Fig. 6. **Simulation model for determination of the amount of heat**

Conclusions

1. Maximum and minimum values for the rotation angles of the collector reflectors are stated. Mathematics coherences for the calculation of the rotation angles of the reflectors are obtained, when the width of the reflectors is not wider than the width of the absorber, $aa \cdot ak\text{-}1 \leq 1$, because it ensures the efficient use of the reflective surface area of the collector.
2. It is recommended to have the rotation interval 10 minutes for the solar collector with reflectors.
3. Comparing the amount of the heat produced by the sun tracking the flat plate solar collector under field conditions with the results obtained using the Matlab-Simulink software, it is stated that at the same radiation intensity the collector with reflectors produces 2.89 times more heat than the collector without reflectors.

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