PIPELINE SEGMENT SQUEEZING BY REACTIVE LOAD DUE TO FLOWING OUT

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Abstract. The stability of the equilibrium configuration for the open segment of the elastic pipeline, conveying inertia fluid, is under consideration. To do this, the fundamental laws of the Euler Dynamics are used, formulated for arbitrary open systems. It is strictly shown that the impact at the end of the pipeline construction by the pressing force and the reactive force of the recoil jet are identical. As a consequence of this identity is shown the possibility of the non-rectilinear equilibrium configuration of the elastic pipeline existence. Such type of static configuration for the open segment of the pipe depends on the boundary conditions and may appear if the speed of the stream reaches to the critical value. The reactive force of the jet for the critical speed of the flow is equal to the critical value of the compressive force at which the rectilinear configuration of the rod system becomes unstable according to the classical scenario. This proves that the mechanisms of stability lose rectilinear configuration of the elastic pipe, conveying inertial flow under the influence at the end of the pressing force and the recoil force of the jet being the same.

Keywords: pipeline stability, non-conservative system.

Introduction

Currently, at the determination of stability conditions of equilibrium configurations of the open pipeline systems the role of the recoil force of the jet flowing out of the pipe remains unclear. Several problems on vibrations and stability of the rectilinear configurations of the pipeline systems are reviewed in [1]. However, the Feodosyev comments for the solutions of these problems do not give clear explanation whether the squeezing of the open pipeline by the recoil force of the jet is the main cause the of stability loss of its straight equilibrium configuration. Or some other physical phenomena lead to the loss of stability of the open pipeline segment?

Uncertainty about the significance of the influence of the recoil force of the jet on the properties of the open pipeline system is reflected in very many studies of stability of the pipeline segment, through which the inertia fluid is flowing. The studies of the deformable straight cantilever pipe vibrations with the inertial fluid flow were presented in literature [2; 3]. In these works the influence of the recoil force of the jet at the pipe face end is absolutely ignored. The validity of this view is questionable, as since 1966 until now, a satisfactory explanation of the paradoxical results of the theoretical modeling of the dynamics of the cantilever pipe, conveying heavy fluid is not offered [1; 4-7]. Applied relevance of the proposed studies is identified by the need to ensure trouble-free operation of pipeline systems in the light of the prospects of mining from the bottom of the seas and oceans, as well as a large-scale operation currently performed in a network of underwater pipelines.

In mechanics, the pipeline systems are considered as elongated structures consisting of hollow thin rods along which a given flow rate is provided due to the energy of external devices to transported inertial flow. Longitudinal flow rate is a parameter, which is affordable to variation, affecting the stability of the pipeline system. The presence of the pumping device provides a nonconservative property to the pipeline systems [1; 4; 7].

Most mathematical models of pipe systems conveying the inertial fluid flow, described in literature, are focused solely on the numerical analysis [7; 8]. There is lack of analytical results in the branches of mechanics, studying vibrations and stability of pipelines with the inertial fluid flow. In this regard, there are inconsistencies and contradictions in the interpretation of the theoretical study of the interaction between an elastic inertia pipe and the inertia flow [4-6; 8; 9].

An open pipeline segment

Consider the model of the pipeline system segment shown in Fig. 1. The kinematics of the system allows only one degree of freedom. So we hope to obtain the transparent analytical results [10]. The configuration of the system is completely determined by the angle value φ . The system is composed of two thin hollow flow guiding rods with the interior channels of the cross section area F. An incompressible inertia fluid with density ρ is supplied in the rods by a pump H.

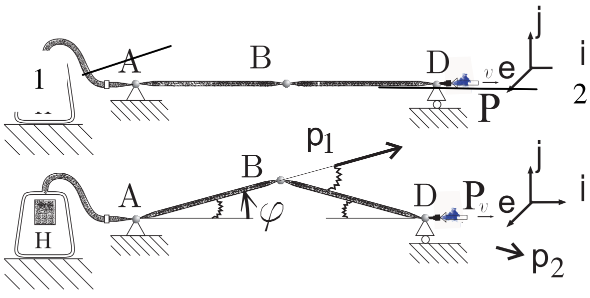

Fig. 1. **Pipeline with fixed direction of fluid leakage**

The rods are closely connected by an elastic cylindrical hinge B. A flow guiding rod of mass m_1 and length l_1 connects the hinges A and B. The axis of the rod coincides with the direction of the mobile unit vector $\mathbf{p}_1$. This rod is pinned in the fixed elastic cylindrical hinge A. A rod, connecting the hinges B and D, is of length l_2 and mass m_2. The end D of this rod is pinned in a movable elastic cylindrical hinge D. The hinge D is allowed to move along the unit vector $\mathbf{i}$ (Fig. 1).

The axis of the rod, connecting the hinges B and D, is directed along the movable unit vector $\mathbf{p}_2$. The hole area in each hinge of the system, through which the liquid enters into the rods, shall be equal to the cross sectional area of the interior channels of the flow guiding rods. Fluid is injected into the system from outside through the hinge A. In the hinge D fluid leaves the rod system along a fixed direction $\mathbf{i}$. A virtual compressive force $\mathbf{P} = -P\mathbf{i}$ is applied to the hinge D. It plays an auxiliary role for the analysis of the pipeline system stability.

The governing relations for the elastic hinges

Moments of the connections have elastic components, orthogonal to the plane of the drawing as well as components to prevent the displacements of the system out of this plane.

$$\mathbf{M}_A = \mathbf{M}_A^* + \mathbf{M}_A^{el}\mathbf{e}, \qquad \mathbf{M}_B = \mathbf{M}_B^* + \mathbf{M}_B^{el}\mathbf{e}, \qquad \mathbf{e} = \mathbf{i} \times \mathbf{j}$$

$$\mathbf{M}_D = \mathbf{M}_D^* + \mathbf{M}_D^{el}\mathbf{e}, \qquad \mathbf{e} \bullet \mathbf{M}_A^* = 0, \qquad \mathbf{e} \bullet \mathbf{M}_B^* = 0, \qquad \mathbf{e} \bullet \mathbf{M}_D^* = 0.$$

Let us denote by μ_A the bending stiffness of the elastic element in the hinge A. For the elastic component of the moment $\mathbf{M}_A$, acting on the rod, connecting the hinges A and B, we use the constitutive relation $\mathbf{M}_A - \mathbf{M}_A^* = -\mu_A \mathbf{i} \times \mathbf{p}_1$. This relation is linear for small deviations of the system from a straight configuration. For the elastic component of the moment $\mathbf{M}_B$, affecting on the rod between the hinges A and B, we use a relation $\mathbf{M}_B - \mathbf{M}_B^* = \mu_B \mathbf{p}_1 \times \mathbf{p}_2$, where μ_B is the bending stiffness of the elastic element in the hinge B. To describe the elastic component of the torque impact on the rod connecting the hinges B and D, from the side of the hinge D, assume the relation $\mathbf{M}_D - \mathbf{M}_D^* = \mu_D \mathbf{p}_2 \times \mathbf{i}$. Here μ_D is the bending stiffness of the elastic element in the hinge D. It is obviously $l_1\mathbf{p}_1 + l_2\mathbf{p}_2 = l_{12}\mathbf{i}$, where, l_{12} is a distance between the hinges A and D. For the moment impacts in the hinges, it is convenient to use the governing relations in the following form:

$$\begin{aligned}&\mathbf{M}_A = \mathbf{M}_A^* + \mu_A \mathbf{p}_1 \times \mathbf{i}, \quad \mathbf{M}_B = \mathbf{M}_B^* + \mu_B l_2^{-1} l_{12} \mathbf{p}_1 \times \mathbf{i}, \qquad \mathbf{M}_A^* \cdot \mathbf{e} = 0, \qquad \mathbf{M}_B^* \cdot \mathbf{e} = 0,\\ &\mathbf{M}_D = \mathbf{M}_D^* - \mu_B l_1 l_2^{-1} \mathbf{p}_1 \times \mathbf{i}, \quad \mathbf{p}_1 \times \mathbf{i} = \mathbf{e}\sin\varphi, \qquad \mathbf{M}_D^* \cdot \mathbf{e} = 0.\end{aligned} \tag{1}$$

Nonconservatism

There is a constant inflow of energy from an external source in the considered system. Assume the pump H provides a constant flow of fluid q in the hinge A. Let us denote by v the fluid speed through the hole with the area F in the hinge A. This speed is related to the fluid flow rate in the hinge A by the relation:

$$q = \rho F v\,. \tag{2}$$

As q is a constant, as well as the cross-sectional areas of the internal channels flow guiding rods, the value of a longitudinal flow speed remains constant with respect to the axis of each rod for any configuration of the deformable system.

The laws of motion for an open system

In accordance to the fundamental laws of the Euler dynamics, formulated for the bodies of arbitrary nature in [11; 12], we can write the following relations

$$\begin{aligned}
&\dot{\mathbf{K}}_1^{AB} = \mathbf{N}_A + \mathbf{N}_B + \mathbf{k}_1^{AB}, \quad ()^{\bullet} = d/dt\\
&\dot{\mathbf{K}}_{2,A}^{AB} = \mathbf{M}_A + \mathbf{M}_B + \mathbf{r}_1 \times \mathbf{N}_B + \mathbf{k}_2^{A,AB}, \quad \mathbf{r}_1 = l_1\mathbf{p}_1,\\
&\dot{\mathbf{K}}_1^{BD} = -\mathbf{M}_B + \mathbf{M}_D + \mathbf{P} + \mathbf{k}_1^{BD}, \quad \mathbf{i}\bullet\mathbf{N}_D = 0,\\
&\dot{\mathbf{K}}_{2,A}^{BD} = \mathbf{M}_A + \mathbf{M}_B - \mathbf{r}_1 \times \mathbf{N}_B + \mathbf{r}_{12} \times \mathbf{N}_D + \mathbf{k}_2^{A,BD}, \quad \mathbf{r}_2 = l_2\mathbf{p}_2,\\
&\dot{\mathbf{K}}_{2,A}^{\Sigma} = \mathbf{M}_A + \mathbf{M}_D + \mathbf{r}_{12} \times \mathbf{N}_D + \mathbf{k}_2^{A,ABD}, \quad \mathbf{r}_{12} = \mathbf{r}_1 + \mathbf{r}_2.
\end{aligned} \tag{3}$$

Vectors $\mathbf{N}_A$, $\mathbf{N}_B$, $\mathbf{N}_D$, $\mathbf{M}_A$, $\mathbf{M}_B$, $\mathbf{M}_D$ in (3) are the force and moment reactions of hinge connections of the rod system. The relations (3) contain the vector measures of motion: $\mathbf{K}_1^{AB}$ that is the rectilinear (or kinetic) momentum of a flow guiding rod with the inertia fluid, connecting the hinges A and B; $\mathbf{K}_{2,A}^{AB}$ is the angular momentum with respect to the point A of this rod; $\mathbf{K}_1^{BD}$ and $\mathbf{K}_{2,A}^{BD}$ are similar values for the rod connecting the hinges B and D; $\mathbf{K}_{2,A}^{\Sigma} = \mathbf{K}_{2,A}^{AB} + \mathbf{K}_{2,A}^{BD}$ is the total angular momentum of the open rod system with respect to the point A.

Reactive terms

The reactive terms are included in (3) because each open rod exchanges the mass with its surrounding. The term $\mathbf{k}_1^{AB}$ is an inflow rate of the rectilinear momentum into the rod connecting the hinges A and B; $\mathbf{k}_2^{A,AB}$ is an inflow rate of the angular momentum with respect to the point A into the rod connecting the hinges A and B; $\mathbf{k}_1^{BD}$ is an inflow rate of the kinetic momentum into the rod connecting the hinges B and D; $\mathbf{k}_2^{A,BD}$ is an inflow rate of the angular momentum with respect to the point A into the rod connecting the hinges B and D, and $\mathbf{k}_2^{A,ABD}$ is an inflow rate of the angular momentum with respect to the point A into the whole rod system. Constitutive relations for reactive impacts on a rod system are obtained on the basis of determination

$$\mathbf{k}_1^{AB} = \lim_{\Delta t \to 0} \frac{\mathbf{K}_1^{in\,AB}(t,\Delta t) - \mathbf{K}_1^{out\,AB}(t,\Delta t)}{\Delta t}\,. \tag{4}$$

In (4) $\mathbf{K}_1^{in\,AB}(t,\Delta t)$ is the rectilinear momentum, entering into the open rod AB together with an inertia fluid during the time Δt; $\mathbf{K}_1^{out\,AB}(t,\Delta t)$ is the rectilinear momentum, leaving the open rod AB during the time Δt due to fluid outflow at the point B.

$$\mathbf{K}_1^{in\,AB}(t,\Delta t) = \Delta m \mathbf{v}_A^s, \quad \mathbf{K}_1^{out\,AB}(t,\Delta t) = \Delta m \mathbf{v}_B^s, \quad \Delta m = q\Delta t\,. \tag{5}$$

In (5) the vector $\mathbf{v}_A^s$ is an absolute velocity of the inertia substance flow element supplied into the open body at the point A; $\mathbf{v}_B^s$ is a similar value for the inertial fluid flow element flowing through the face end B. For v_A^s and v_B^s we have

$$\mathbf{v}_A^s = v\mathbf{p}_1, \quad \mathbf{v}_B^s = v\mathbf{p}_1 + \mathbf{\Omega}_1 \times \mathbf{r}_1 .$$

Here $\mathbf{\Omega}_1 = \Omega_1 \mathbf{e}$ is the angular velocity of the rod connecting the hinges A and B. Using (4) we find

$$\mathbf{k}_1^{AB} = -q\mathbf{\Omega}_1 \times \mathbf{r}_1 . \tag{6}$$

The rectilinear momentum, leaving the rod between the hinges A and B, enters into the rod connecting the hinges B and D. Thus,

$$\mathbf{K}_1^{in\,BD} = \Delta m \mathbf{v}_B^s, \quad \mathbf{K}_1^{out\,BD} = \Delta m \mathbf{v}_D^s . \tag{7}$$

The vector $\mathbf{v}_D^s = \mathbf{v}_B + \mathbf{\Omega}_2 \times \mathbf{r}_2 + v\mathbf{i}$ is the total flow velocity at the point D, the vector $\mathbf{\Omega}_2 = \Omega_2 \mathbf{e}$ is the angular velocity of the rod between the hinges B and D. So for $\mathbf{k}_1^{BD}$ the definition (4) gives

$$\mathbf{k}_1^{BD} = qv\mathbf{p}_1 - q(\mathbf{\Omega}_2 \times \mathbf{r}_2 + v\mathbf{i}) . \tag{8}$$

Similarly, for the rate of angular momentum influx we obtain

$$\mathbf{k}_2^{A,AB} = \lim_{\Delta t \to 0} \frac{\mathbf{K}_2^{in\,AB}(t,\Delta t) - \mathbf{K}_2^{out\,AB}(t,\Delta t)}{\Delta t} = -ql_1^2 \mathbf{\Omega}_1 \tag{9}$$

$$\mathbf{k}_2^{A,BD} = \lim_{\Delta t \to 0} \frac{\mathbf{K}_2^{in\,BD}(t,\Delta t) - \mathbf{K}_2^{out\,BD}(t,\Delta t)}{\Delta t} = ql_1^2 \mathbf{\Omega}_1 . \tag{10}$$

For the selected type of inertial fluid leakage from the pipeline $\mathbf{k}_2^{A,ABD} \equiv \mathbf{0}$.

Let us investigate the existence of the static solutions for (3). For such a solution the conditions $\dot{\varphi} = 0$ and $v \neq 0$ have to be satisfied. In this case, according to (6) and (9), we obtain $\mathbf{k}_1^{AB} = \mathbf{0}$, $\mathbf{k}_2^{A,AB} = \mathbf{0}$. As the left part of each equation in (3) is equal to zero, we can obtain the following relation

$$\mathbf{M}_A + \mathbf{M}_B + \frac{\mathbf{r}_1 \times (\mathbf{i} \times \mathbf{M}_A + \mathbf{i} \times \mathbf{M}_D + \mathbf{i} \times \mathbf{k}_2^{A,ABD})}{\mathbf{i} \bullet \mathbf{r}_{12}} = -\mathbf{r}_1 \times (\mathbf{P} + \mathbf{k}_1^{BD}) . \tag{11}$$

Using (1) and (7), we get a scalar equation from (11) as follows

$$0 = \{\mu_A + \mu_B l_2^{-1} l_{12} - (\mathbf{i} \bullet \mathbf{r}_{12})^{-1} l_1 \{\mu_A - \mu_D l_1 l_2^{-1}\} \mathbf{p}_1 \bullet \mathbf{i} - (P + qv) l_1\} \sin\varphi . \tag{12}$$

A rectilinear configuration corresponds to the trivial solution of equation (12), such as $\sin\varphi = 0$. Let us verify the nontrivial case, if the term in the braces is equal to zero.
It is easy to see that

$$\mathbf{p}_1 \bullet \mathbf{i} = \cos\varphi, \quad l_{12} = \mathbf{i} \bullet \mathbf{r}_{12} = l_1 \cos\varphi + l_2 \mathbf{i} \bullet \mathbf{p}_2 .$$

If we denote $\mathbf{i} \bullet \mathbf{p}_2 = \cos\psi$, it is geometrically obvious, that $\sin\psi = l_1 l_2^{-1} \sin\varphi$.

Thus, $\cos\psi = l_2^{-1}\sqrt{l_2^2 - l_1^2 \sin^2\varphi}$. As a result, we get the relationship, connecting the value of φ and the value of the sum, composed of the auxiliary face end squeezing force P and the recoil force of the jet qv. The sum of both forces ensures the existence of the non-rectilinear equilibrium configuration of the considered open system in case if the following is true

$$\begin{aligned} &P + qv = \frac{\mu_A}{l_1} + \frac{\mu_B l_{12}}{l_1 l_2} + \{\mu_D l_1 - \mu_A l_2\} \frac{\cos\varphi}{l_2 l_{12}} . \\ &l_{12} = l_1 \cos\varphi + \sqrt{l_2^2 - l_1^2 \sin^2\varphi} \end{aligned} \tag{13}$$

It is evident from (13), that the impact of the auxiliary compressing force P and the impact of the recoil force of the jet qv are identical.

The impact of the compressive force on the stability of a rod structure is studied quite well. If one assumes $P = 0$, the open system remains under the compression by the jet recoil force qv, so ignoring of this phenomenon is inadmissible.

Let us evaluate the solution of (12) for $l_1 = l_2 = l/2$ and small values of φ. Suppose that $\cos\varphi \approx 1 - \varphi^2/2$ and $\sin\varphi \approx \varphi$. In this case it follows from (13)

$$\varphi_{st}^2 = \frac{\mu_A + 4\mu_B + \mu_D - (P + qv)l}{2\mu_B}. \tag{14}$$

It is obvious, that for small angles φ_{st} there exist at least two static configurations of the system (Fig. 1) for which (13) is correct.

Conclusions

The solvability of (13) proves the equivalence of the compression effect on the open pipeline system due to the squeezing by the canonical force $\mathbf{P}$ and the recoil force of the jet qv. It follows that the rectilinear equilibrium configurations of the pipeline system with an inertia flow inside (Fig. 1) loses the stability in accordance to the classical scenario in case if the recoil force of the jet exceeds the critical value of the corresponding compressive face end force. This conclusion may be extended on an open segment of any pipeline system. It does not deny the existence of the dynamical scenarios for the loss of stability of a pipe system. But the critical squeezing by the recoil force at the face end of the open segment has to be considered as one of the main scenarios. Up to now, the stability loss mechanism of the pipeline with the flow of incompressible inertia fluid based on the squeezing of the pipeline by the jet recoil force has not been discussed widely in literature because of lack of the transparent and understandable analytical results.

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