MODEL OF PRICE VOLATILITY RISK MANAGEMENT FOR FIRM WITH PERFECTLY STRUCTURED TECHNOLOGY

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Abstract. In this paper, a multi-product firm is covered, and an original model for management of the risk that arises from product price volatility is presented. The model is applicable in cases, when the company management without significant cost may vary the quantities of individual products within a given output structure. According to the classical paradigm of risk analysis by Daniel Bernoulli, with respect to stochastic profit, every plan of output is characterized by two criteria – expected profit and standard deviation of profit. In order to characterize the strategies of management, the concept of Bernoulli set is introduced in this paper. Ideas of Markowitz and Freund are applied to the firm TPF (technology, production, finances) model, which consider in the holistic approach the technology, production and finances of the firm.

Keywords: price volatility, risk management, TPF model, Bernoulli set, Pareto frontier, maximal efficient risk.

Introduction

We live in a world where necessity and randomness exist as dialectical unity of contradictions. From a viewpoint of a decision-maker, the uncertainties of future developments are various. Extreme uncertainty, when the manager does not know the whole set of possible economic states, but is aware only of the subset of possible states. Uncertainty, when the manager knows the set of possible economic states, but is ignorant about the probabilities of individual states. It is not possible to measure this uncertainty. Uncertainty, which is measurable: the manager knows the probability distribution in a set of economic states.

Our research refers to the case of uncertainty that is measurable. We classify commercial operation as risk operation, if it may have more than one outcome and if at least two of the outcomes of this operation do not have the same level of utility or, in other words, are not indifferent to the person who makes the decisions.

Micro-analysis of risk always takes into account the person who assumes the risk, to whom this risk refers, and who is concerned about the outcome of the risky operation. The risks of firms are varied: technological and production risks, risks in product markets, risks in resource markets, investment and financing risks. At scientific conferences, theses have been expressed that it is most dangerous for a firm to lose its reputation. Indeed, the confidence crisis in the world creates an avalanche of risks without any precedents.

Research on decision-making in risky circumstances is founded on the fundamental paradigm of risk management: quantitative estimate of commercial operation risk is possible only if probability distribution exists in a set of potential situations corresponding to each alternative strategy. Risk management seeks for compromise between the gains from risk decrease and the cost of risk decrease. We shall remember that all decisions must be made *ex ante* – before each specific situation sets in and uncertainty disappears. If the manager of a company is able to forecast and manage risks, he will earn abnormal profit. Theory of economics recognizes the role of intuition and presentiment in decision-making. Alfred Marshall in his famous book "Principles of Economics" published in 1890 ascertains that the manager makes decisions not so much by basing on knowledge as on well-developed instinct, intuition, presentiment.

As the problem of decision-making is analyzed in relation to specific commercial operation, the person, who makes decisions, is extremely interested in obtaining a relevant information about a given problem. One of legal sources of information is comprehensive informative analysis of a mathematical model, which should be carried out with the help of mathematical analysis and computer software. Nowadays, operation research as a management science undergoes principal transformation.

In the paper, a multi-product firm is covered, and an original model for management of the risk that arises from product price volatility is presented. The model is applicable in cases, when the company management without significant cost may vary the quantities of individual products within a given output structure. For example, when planning production and by taking into account the expected prices of products, rural entrepreneurship may decrease or increase the output quantities of individual products, which we observe in practice. Asthe planned total cost in calculated, the manager does not face uncertainty, since the present prices of resources are known. It is known that technologies are persistent over time. Therefore, the most typical producer's risks relate to volatility of the product prices in the future. For illustration, we publish the vector of average values of the agricultural product stochastic prices vector, the vector of variation coefficients, and the matrix of variation coefficients.

In this research, we regard technologies that have been implemented in a firm, machinery installed and the quality of technological management as invariable and concentrate on risks, which company incurs due to price volatility. In contemporary business, a common form of organization is corporation, which is characterized by dispersed property rights. Owners are alienated from their property, and corporation is being managed by the techno structure, which is not homogenous. Therefore, gaining understanding of the motivation that persons involved in decision-making have is difficult. Decision-making in groups therefore is one of the most obscure issues, among which the paradox of democratic paralysis by M. de Condorcet, theorem of dictator by K. J. Arrow, theorem about the role of extremal restriction in consolidating individual preferences by A. K. Sen, P. K. Pattanaik. A notion about the newest research in the field of collective decision-making may be formed, for example, by getting acquainted with the book [1]. In our research, where the problems of operation research, economics of the firm and risk engineering meet, an important concept is sovereign owner of the form – the person who makes decisions at a firm. We categorically avoid talking about decision-making in groups, therefore we will consider companies, which are owned by one owner. The owner is a person who makes decisions at his firm with an aim of utility maximization. According to contemporary paradigm of microeconomics, utility of the owner is measureable in terms of the present value of the company's future cash flows, where the time value of money is expressed as the opportunity cost of capital. According to the simplest interpretation, an owner wants to maximize the profit. With such an approach this paper will be useful for understanding of the economics of Latvia's small and medium – sized companies, because at such companies the owner manages his own property.

Theoretical background and methods

The classical paradigm of risk analysis by Daniel Bernoulli (1700-1782), the idea about diversification of investments of Harry Max Markowitz (born 1927), the concept of Pareto-efficient strategy, defined by the classic of microeconomics and socioeconomics Vilfredo Federico Damaso Pareto (1848-1923) form the theoretical background of the research.

In the work by Daniel Bernoulli "*Specimen theoriae novae de mensura sortis*" (English translation in Bernoulli D., 1954 – "Exposition of a New Theory on the Measurement of Risk." – Econometrica 22) the most important concepts of modern mathematical economics are provided. Daniel Bernoulli introduced a paradigm according to which the agent of economics – the person who makes decisions, measures the stochastic outcome of economic operation – win *W* in terms of the expected value E(W) and standard deviation SD(W). Besides, the agent regards the expected value E(W) as good, and standard deviation of prize SD(W) – as a measure of risk. In psychology, it has been proved that most economic agents are risk averse and regard the standard deviation of prize as bad. How does the economic agent, which strives towards the greatest possible expected prize E(W), but is afraid of too high a risk SD(W), behave?

Contradiction between greediness and fear of the economic agent Daniel Bernoulli solves in a completely innovatory manner, by formulating the fundamental principle of modern economics: the agent measures prize W – the stochastic result of economic operation – according to his individual utility function u = u[E(W), SD(W)].

The employee of RAND Corporation Harry Markowitz made use of the established feature of variance "$D(aW_1 + bW_2) = a^2 D(W_1) + 2ab\, cov(W_1, W_2) + b^2 D(W_2);\ \forall\ a, b \in \mathbf{R}$" and in his paper "Markowitz H. M. Portfolio Selection. – Journal of Finance 7, 1952" published an idea about possibilities for risk decrease by means of purposeful formation of securities portfolio.

The idea of investment diversification earned huge responsiveness and in 1990, Harry Markowitz earned a Nobel Prize in economics. The concept of Pareto-efficiency, provided by Vilfredo Pareto, is applied in analysis of diversification results [2; 3; 4].

The idea of the Markowitz model was extended and modified by Freund [5] designing the first linear programming model with risk in agriculture. Freund (1956) wrote: ...probabilities distribution of net revenue of a unit level of a process can be developed from practically any assumption about various components that enter into net revenue,..., for example, the price of outputs of the process are subject to random fluctuation while all quantities and prices of the input are at fixed, known levels. The Freund-Markowitz model has been used in a great number of studies. In the book [6] the present results in this field of study have been thoroughly presented.

Inspired by ideas of Markowitz and Freund, Jaunzems [7] offers a general model of assets portfolio structure "Assets Portfolio Selection".

In this paper, in the context of "Assets Portfolio Selection," the problem of the firm's product price risk volatility management has been reviewed. The research methods are microeconomic and mathematic analysis, operations research and mathematical modelling. Research on Bernoulli set of a virtual firm has been carried out with a simulation method, which allows testing the efficiency of risk management strategies empirically. For simulation, a matrix model of a multi-product firm TPF (technology, production, finances), which integrates in a single system firm technology, production and finances, has been applied.

With a help of the TPF model, the firm's technological, production and financial limitations and goals may be modelled. Let us emphasize the prominent historical significance of the book "*Р. Дж. Д. Аллен. Математическая экономия. – Москва: Издательство иностранной литературы. 1963*" in education of engineers-economists in Soviet Latvia. By critically evaluating the possibilities of traditional marginal analysis, the classic of mathematical economics, Professor of the London School of Economics Roy George Douglas Allen (1906-1983) emphasizes the advantages of a TPF model [8]: "The basis of the linear programming approach to the theory of the firm is the specification of a technology in a way both more specific and more detailed than the production function of marginal analysis. Indeed, linear programming goes out of its way to stress the technical side of the decisions made by the firm. It starts from assumption that the firm considers only a limited number of separate technical processes of production. Put in this way, the linear programming approach seems very well adapted for application to decision-taking at level of the firm. It provides, trough emphasis on technology, just the link required between the problems of interest to the economist and those which engage the attention of entrepreneur and engineers."

We agree with Joseph Alois Schumpeter, who in his book [9] stated that it is more productive to study the economics of a firm, by analyzing the operations of a virtual firm than by analyzing a practical real-life example. The reason for this statement is the fact that in real life, the significant relationships and principles are influenced and overshadowed by various very manifold side-effects and occurrences. Regularly, research by means of simulation in the era of computers gains an ever increasing role. Laboratory experiments with simulated situations allow discovering regularities, that may be generalized as laws of economics, but which it is sometimes difficult to prove with analytical methods like theorems.

Let us note that with respect to stochastic value-added each output of a multi-product firm is characterized by the expected value-added and standard deviation of value-added. The offered model with minor modifications is applicable also for management of value-added risks.

Well-known and recognized assertion is that uncertainty and risk are essential features of agricultural production. The wide studies of scientific literature [10-15] attest that risks of agricultural production related to food price volatility are widely studied – for example, Moschini and Hennesy (1999), Meuwissen, Hardaker, Huirne and Dijkhuizen (2001), Ladányi (2007), Sulewski and Kloczko-Gajewska (2014). The paper of Jesús Antón (2008) represents the point of view of Trade and Agriculture Directorate OECD-OCDE and discusses the main issues and driving forces of government policies in the area of risk management in agriculture (a holistic approach).

Results and discussion

1. Volatility of prices in Latvia's market for agricultural produce

Gilbert and Morgan [15] formulated four most fundamental economical food price volatility factors: an increase in the variance of demand shocks; an increase in the variance of supply shocks; decline in the elasticity of demand; decline in the elasticity of supply. These factors can also be identified in Latvia's market for agricultural goods. In this paper, however, we do not set a goal to analyze the influence that the factors formulated by Gilbert and Morgan exert on risks of agricultural production. To ascertain that the diversification of risks, which is offered in this paper, is motivated in Latvia's agricultural production, we performed calculations with time series (Table 1) of 10 popular agricultural goods retail prices and found that small positive or even negative correlation exists between the prices of different products. We made use of the Statistics Database "Consumer prices" of the Central Statistical Bureau of the Republic of Latvia. The database "Consumer prices" contains weighted arithmetical average prices of selected consumer goods, which may be easily grouped and described according to common characteristics, mainly of food products and selected non-food commodities. The main aim of the price registration is evaluation of the price changes during a certain period of time with an aim to calculate CPI. Prices have been shown including all taxes.

Table 1

Average retail prices of selected commodity (euro per 1 kg, if other – specified)

Commodity	2010	2011	2012	2013	2014	2015
Beef	3.78	4.14	4.57	4.78	4.75	4.79
Pork	3.37	3.39	3.53	3.70	3.71	3.72
Milk, per 1 litre	0.73	0.85	0.84	0.84	0.89	0.76
Potatoes	0.34	0.47	0.33	0.40	0.39	0.36
Cabbage, fresh	0.44	0.57	0.34	0.46	0.46	0.46
Carrots	0.48	0.51	0.55	0.60	0.52	0.63
Onions	0.64	0.58	0.43	0.55	0.59	0.53
Apples	0.80	1.10	1.04	1.10	1.03	1.06
Honey	7.48	7.87	7.74	8.08	8.67	9.49
Eggs, per 10 pieces	1.34	1.29	1.59	1.48	1.40	1.37

Source: http://www.csb.gov.lv/en/dati/statistics-database-30501.html (viewed on: 29.02.2016).

Let us number the food products from 1 to 10: (1) beef; (2) pork; (3) milk; (4) potatoes; (5) cabbage, fresh; (6) carrots; (7) onions; (8) apples; (9) honey; (10) eggs.

In Table 2, the matrix of the product price correlation coefficients, the vector of average prices and the vector of variation coefficients is provided.

Table 2

Correlation matrix of 10 products' average retail prices; average prices; variation coefficients (Latvia, 2010-2015)

Product No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	1	-	-	-	-	-	-	-	-	-
(2)	0.94	1	-	-	-	-	-	-	-	-
(3)	0.48	0.31	1	-	-	-	-	-	-	-
(4)	-0.04	-0.12	0.49	1	-	-	-	-	-	-
(5)	-0.23	-0.18	0.12	0.90	1	-	-	-	-	-
(6)	0.79	0.76	-0.05	-0.12	-0.15	1	-	-	-	-
(7)	-0.53	-0.28	-0.26	0.32	0.61	-0.51	1	-	-	-
(8)	0.71	0.51	0.66	0.55	0.26	0.60	-0.51	1	-	-
(9)	0.69	0.77	0.00	-0.01	0.13	0.70	-0.10	0.41	1	-
(10)	0.50	0.34	0.29	-0.54	-0.84	0.33	-0.81	0.22	-0.14	1
Average prices	4.47	3.57	0.82	0.38	0.46	0.55	0.55	1.02	8.22	1.41
Variation coefficients	0.09	0.04	0.07	0.12	0.15	0.10	0.12	0.10	0.08	0.07

Source: authors' calculation used Table 1.

2. Assets portfolio selection model

Let us discuss the basic ideas of the model "Assets portfolio selection" [7].

Let us examine n assets, the quantities of which $x_1, x_2, \dots, x_n$ are expressed in certain units of measurement. We regard the prize which the owner of the asset will obtain from a unit of i-th asset $x_i = 1$ in a certain market during the next period in the future as random amount W_i with an expected win $E(W_i) = \mu_i$, variance $D(W_i) = \sigma_i$, and standard deviation $SD(W_i) = \sigma_i$; i = 1, 2, ... , n.

n-dimensional probability distribution $(W_1, W_2, \dots, W_n)$ of random amount is given, therefore, we can calculate covariance $cov(W_i, W_j) = \sigma_{i\,j}$ and correlation $cor(W_i, W_j) = \rho_{i\,j}$ between stochastic wins W_i, W_j ; i, j = 1, 2, ... , n.

It is convenient to apply vector and matrix denotations.

We introduce the vector of the expected values $M := (\mu_1\ \mu_2 \dots \mu_n) \in \mathbf{R}^n$.

The covariance and correlation matrices of n-dimensional random number $(W_1, W_2, \dots, W_n)$ we will denote $cov(W_1, W_2, \dots, W_n) = (\sigma_{i\,j}) \in \mathbf{R}^{n,\,n}$, $cor(W_1, W_2, \dots, W_n) = (\rho_{i\,j}) \in \mathbf{R}^{n,\,n}$, respectively.

In agreement with the axiom of linearity the stochastic win of $X = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n$ equals $W(X) := x_1 W_1 + x_2 W_2 + \dots + x_n W_n$.

The expected value of stochastic win W(X) can be expressed as the linear function of X in the form of the vector scalar product $E(W(X)) = M \cdot X$, and variance can be expressed as the square function of X:

$D(W(X)) = X\ cov(W_1, W_2, \dots, W_n) \cdot X$.

Let us assume that the owner of assets may choose the portfolio of assets X out of set $\mathbf{X} \subset \mathbf{R}^n$.

We will interpret the choice of the assets portfolio X as the strategy chosen by the owner of assets.

Function *B*, which attaches a pair of numbers $(SD(W(X)), E(W(X))) \in \mathbf{R}^2$ to strategy $X \in \mathbf{X}$ we name as Bernoulli operator and write $B(X) = (SD(W(X)), E(W(X)))$. The two-dimensional depiction in a (σ, μ)-plane of a set of strategies X we denote with **B** and name as Bernoulli set.

Therefore, $\mathbf{B} := \{(SD(W(X)), E(W(X))) \mid X \in \mathbf{X}\} = B(\mathbf{X}) \subset \mathbf{R}^2$.

It is easy to calculate the Bernoulli set **B** and to depict it graphically.

Ordinate $\mu^{max}(\sigma)$, which corresponds to the abscissa σ of the Bernoulli set upper border $(\sigma, \mu^{max}(\sigma))$, can be expressed in form: $\mu^{max}(\sigma) = max\ \{E(W(X)) \mid X \in \mathbf{X}, SD(W(X)) = \sigma\}$ with Lagrange multiplier $\lambda^{max}(\sigma)$.

Ordinate $\mu^{min}(\sigma)$, which corresponds to the abscissa σ of the Bernoulli set lower border $(\sigma, \mu^{min}(\sigma))$, can be expressed in form: $\mu^{min}(\sigma) = min\ \{E(W(X)) \mid X \in \mathbf{X}, SD(W(X)) = \sigma\}$ with Lagrange multiplier $\lambda^{min}(\sigma)$.

Lagrange multipliers $\lambda^{max}(\sigma)$, $\lambda^{min}(\sigma)$ can be interpreted as marginal maximal expected win with respect to risk, marginal minimal expected win with respect to risk:

$$\frac{d\mu^{max}(\sigma)}{d\sigma} = \lambda^{max}(\sigma),\ \frac{d\mu^{min}(\sigma)}{d\sigma} = \lambda^{min}(\sigma), \text{ respectively.}$$

The bi-criterial (σ, μ) Pareto frontier **P** of the Bernoulli set **B** is characterized by inequality $\lambda^{max}(\sigma) > 0$.

How is diversification of risks carried out in practice? The owner of assets will choose that point P* of Pareto frontier **P**, which maximizes his utility. Afterwards, he will choose the most suitable strategy X* in the inverse image of Pareto frontier's point P* under Bernoulli operator: $X^* \in B^{-1}(P^*) \subset \mathbf{X}$. Sufficiently high value of $\mu(X^*)$, which is in balance with sufficiently low value of $\sigma(X^*)$, lets us to hope for sufficiently high win from a portfolio of assets and to a certain degree avoid the volatility risk of win W(X*).

3. Behavior of producer in certainty circumstances

At the firm "Franky," owned by Mr. Frank, technologies **T** are implemented and machinery **M** has been installed, which allows during a certain period of time, by utilizing the vector or resources

$X = (x_1\ x_2\ ...\ x_n) \in \mathbf{X}$ to produce multi-product $Y = (y_1\ y_2\ ...\ y_m) \in \mathbf{Y}$. Utilization of resources $x_1, x_2, ... , x_n$ and quantities of products $y_1, y_2, ... , y_m$ are expressed in certain units of measurement.

Set $\mathbf{X} \subset \mathbf{R}^n$ is a set of available inputs, and set $\mathbf{Y} \subset \mathbf{R}^m$ is a set of available outputs.

The process of production represents the transformation of the resources vector X into the products vector Y.

We will particularly emphasize that utilization of resources $X \in \mathbf{X}$ under conditions of uncertainty unequivocally determines the output $Y \in \mathbf{Y}$**,** and, therefore, the relationship $X \rightarrow Y$ is functional, determined by technology **T**. Furthermore, the same output $Y \in \mathbf{Y}$, generally speaking, technology **T** lets to obtain with various inputs of resources. Mutual substitutability of resources, by keeping the output constant under conditions of *ceteris paribus*, is represented by technical rate of substitution. The vectors of resource utilization, which correspond to the output $Y \in \mathbf{Y}$ for a subset of **X** that we will denote by **X**(Y). Compliance $Y \rightarrow \mathbf{X}(Y) \subset \mathbf{X}$ is correspondence.

The vector of resources X, which are necessary for production of the output Y, the owner of the firm buys in resource markets, where prices of various resources may form in various ways. The behavior of the producer is rational, therefore, the producer, for the purpose of producing the output Y, always buys the cheapest vector of resources in a set of resources' vector correspondence **X**(Y). Of course, cases are possible when such "cheaper vectors" are several. Still, we will not sin a lot in relation to business practice, if we will assume that such a resource vector is only one. Let us denote this resources vector by X(Y). Having purchased the vector of resources X(Y), Mr. Frank incurs the input cost with respect to the output TC(X(Y)).

Mr. Frank sells the product Y in various product markets, where prices of different products may form in manifold ways. Having sold the vector of products Y, Mr. Frank earns the output revenue TR(Y).

Profit is revenue from product sales minus the cost of corresponding resource inputs. We will denote the profit earned by the firm owner with a Greek letter π. Then, $\pi := TR(Y) – TC(X(Y))$.

Profit is total revenue with respect to output minus total cost with respect to the corresponding input. Net profit is profit minus direct taxes.

Revenue is influenced by prices and amounts of products sold by the firm. Costs are formed as a result of quantities and prices of the resources supplied, technology, machinery, and the quality of management. Organization forms of markets (perfect competition, oligopoly, oligopsony, monopoly, monopsony etc.). The prices and sales quantities of resources and products are also influenced by the state intervention in the market by means of taxes and subsidies as well as manifold laws and other legal regulations.

When studying the behavior of the producer, in economic theory, one builds on postulate, that the owner of the firm choses such a production plan $Y \in \mathbf{Y}$, which maximizes the utility of the firm's owner. The considerations of the firm's owner may be diverse; in microeconomics of neoclassical synthesis it is assumed that the owner of the firm maximizes the profit $\pi = TR(Y) - TC(X(Y))$.

By correcting the classical approach, we will purposefully note that Mr. Frank makes the management decision about the output Y and buys a vector of resources in present – at the moment of time t_1, but sells the vector of products in future – at the moment of time t_2. Therefore, Mr. Frank incurs the cost TC(X(Y)) before he earns the revenue TR(Y) and, strictly speaking, the profit must evaluated, by taking into account the factor of time and time value of money: $\pi(t_2) = TR(Y, t_2) - (1 + i)^{\tau} \cdot TC(X, t_1)$, $\tau := t_2 - t_1$, where i is interest rate – a measure of time value of money in financial markets.

4. Producer's behavior under conditions of risk related to profit. The principle of Pareto-Bernoulli in a game of measurable uncertainty

The firm "Franky" functions in certain time at a certain place. The firm is located in an environment, the many-sidedness of which is characterized by abbreviation PESTILB environment (political, economic, social, technological, international, legal, bio-environmental). Due to the changes in business environment, profit of the firm is not certain in relation to the chosen vector of output $Y \in \mathbf{Y}$**,** and Mr. Frank incurs risks of the profit. We interpret the choice of alternative production strategy under conditions of uncertainty as Mr. Frank game with objective states of nature, in which also the components of the existing surrounding environment represented by abbreviation PESTILB are recognized as players. The function utility is not attached to these players and their action is regarded as element. In game theory, term "element" (Greek *stoicheion*) is used to denote phenomenon, the action of which is expressed as *force majeure*. Profit is the win of the manager.

When the manager chooses the production strategy and carries out appropriate activities, the business environment PESTILB concretizes and influences the firm's profit. As the decision is made, the manager does not know certainly what the business environment exactly will be like in the future, therefore, in compliance with the paradigm of risk management, we will regard profit as a random amount Π. According to (μ, σ)-paradigm that was invented by Daniel Bernoulli and is widely used in the decision-making theory and practice, the manager evaluates the stochastic outcome of his decision in terms of the expected profit E(Π) and standard deviation SD(Π). The manager considers the expected profit E(Π) as good and standard deviation SD(Π) – as a measure of risk and bad. Thus, the manager describes the stochastic outcome $\Pi = \Pi(Y)$ of each strategy $Y \in \mathbf{Y}$ as a pair of indicators $\sigma(\Pi) := SD(\Pi)$, $\mu(\Pi) := E(\Pi)$, that can be depicted geometrically as a point in (σ, μ)-plane.

According to the general concept mentioned above, we name the function, which attaches a pair of numbers $(\sigma(\Pi), \mu(\Pi)) \in \mathbf{R}^2$ to strategy $Y \in \mathbf{Y}$, a Bernoulli operator, and write $B(Y) = (\sigma(\Pi), \mu(\Pi))$. Two-dimensional representation (σ, μ)-in a plane of strategy set **Y** is denoted by **B** and is named as Bernoulli set.

Thus $\mathbf{B} := \{(\sigma(\Pi), \mu(\Pi)) \mid Y \in \mathbf{Y}\} = B(\mathbf{Y}) \subset \mathbf{R}^2$.

The central concept of bi-criterial (μ, σ) problem analysis is Pareto efficient (Pareto optimal) strategy.

Definition. Strategy $Y^P \in \mathbf{Y}$ is (σ , μ) - Pareto efficient, if no other strategy $Y \in \mathbf{Y}$ exists, such that $\sigma(\Pi) \leq \sigma(\Pi^P)$, $\mu(\Pi) \geq \mu(\Pi^P)$, and at least one of those two inequalities hold in strict sense. This statement can be made differently. Strategy $Y^P \in \mathbf{Y}$ is (σ, μ) - Pareto efficient, if in respect to each strategy $Y \in \mathbf{Y}$, for which $\sigma(\Pi) < \sigma(\Pi^P)$, inequality $\mu(\Pi) < \mu(\Pi^P)$ holds; and in respect to each strategy $Y \in \mathbf{Y}$, for which $\mu(\Pi) > \mu(\Pi^P)$, inequality holds $\sigma(\Pi) > \sigma(\Pi^P)$.

The set of Pareto efficient strategies is denoted by $\mathbf{Y^P}$. The two-dimensional depiction (σ, μ) – in plane of the set of Pareto efficient strategies $\mathbf{Y^P}$ is denoted by **P** and named a Pareto frontier.

Thus $\mathbf{P} := \{(\sigma(\Pi), \mu(\Pi)) \mid Y \in \mathbf{Y^P}\} = B(\mathbf{Y^P})$, $\mathbf{P} \subset \mathbf{B} \subset \mathbf{R}^2$.

A rational manager will choose the strategy of action only among Pareto efficient strategies. Therefore, the main goal of bi-criterial (μ, σ) informative analysis of the game is to determine the set of Pareto efficient strategies $\mathbf{Y^P} \subset \mathbf{Y}$. The manager will choose that point P* on Pareto frontier **P**, which maximizes his utility. Afterwards, he will choose the most appropriate strategy Y* in the inverse image of Pareto frontier's point P* under Bernoulli operator: $Y^* \in B^{-1}(P^*) \subset \mathbf{Y}$.

Note. Jaunzems [16] offers an extension of Bernoulli concept that is easily applicable in practice. By involving such concepts as utility gain and utility loss in analysis of a problem, we obtain new information about the stochastic win form a strategy. Six quantitative criteria are recommended for complex evaluation of a strategy in practice. These criteria are the expected utility of an outcome from an activity; the standard deviation of utility, expected utility gain; standard deviation of utility gain; expected utility loss; standard deviation of utility loss.

5. Multi-product firm TPF model with perfectly structured technology. The diversification of profit risk

As practice attests, technologies are quite persistent over time. The Nobel prize winner in economics Wassily Leontief constructed an input-output model, by basing on this feature. In this study, we will regard the technologies implemented **T**, machinery installed **M** and the quality of technological management of the firm "Franky" as permanent. We will concentrate on risks which the firm incurs due to volatility of prices. Our goal: to develop an original model for management of the product price volatility risk that illustrates the behavior of the producer. In order to facilitate the presentation of this topic, we will build on the matrix model TPF (technology, production, finances). TPF model integrates the firm's technology, production and finance as a united system. With the help of this model, it is possible to model technological, production and financial limitations and goals of a firm. In the entry "*Матричный техпромфинплан предприятия*" of the book "*Экономико-математический словарь.– Москва: Научное издательство "Большая Российская энциклопедия". 2003. 688 с.*" that is rich with content, the description of the TPF model is provided. The reader can encounter the firms' TPF models, that have been elaborated in detail, in special literature.

Let us construct a model, which represents the main and basic causal relationships, though let us not strive towards absolute generality, which is not possible due to limited volume of this paper. Let us consider a virtual firm with perfectly structured technology. The authors have applied such a model for experimental research of various problems of the firm's economics under laboratory conditions, for example, for bi-criterial (value-added, profit) analysis of the multi-product firm output [17].

TPF model of an abstract firm "Franky".

Let us assume that the firm "Franky" owned by Mr. Frank functions in a market of perfect competition for resources as well as in a market of perfect competition for its output. The firm 'Franky" produces five types of products. The quantities of the output produced are measured in natural units of measurement.

We will denote the vector of gross output by $Z = (z_1\ z_2\ z_3\ z_4\ z_5)^T$.

Final product intended for sale equals $Y = (y_1\ y_2\ y_3\ y_4\ y_5)^T$.

It is given that the capacity of the firm "Franky" is limited: $Z \le Z_0$.

Two types of materials purchased from other firms (utilization is denoted by x_1, x_2, respectively) and one type of outsourced service (x_3), labor with high and low qualification contracted in labor market (x_4, x_5) are used in production. Two types of machinery, which are installed at the firm "Franky", are used in production, and the utilization of machine time equals (x_6, x_7).

Therefore, the vector of resource utilization equals $X = (x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7)^T$. Quantities of resources are expressed in certain natural units of measurement. The amount of labor is measured in labor hours; the use of machinery – in machine hours.

Let us assume that the firm's technology is perfectly structured. It means that the utilization of all t resources X depending on gross output Z as well as the usage of self-produced products can be calculated with the help of the (12×5) - matrix of technological coefficients or direct input coefficients (matrix of technologies, in another words). The direct input coefficients at the firm "Franky," in natural units, are given in Table 3. Each column shows resource inputs that correspond to one output unit of the respective product type.

Technology matrix (Table 3) is divided in four sub-matrices:

(5×5)-matrix A, which alike the input-output model characterizes the self-produced product as the inputs of resources at a firm itself;

(3×5)-matrix ER (external resources), which shows the inputs of materials and services acquired externally per unit of each product type;

(2×5)-matrix MT (machine time), which shows the inputs of machinery time per unit of each product type;

(2×5)-matrix LT (labor time), which shows the inputs of labor time per unit of each product type.

Primary variable is the vector of gross output Z. Depending on the gross output Z we calculate: final product $Y = (I - A)\ Z$, utilization of external resources $E = (ER)\ Z$, utilization of machine time $M = (MT)\ Z$, utilization of labor time $L = (LT)\ Z$.

The vector of prices of external resources PER is given, the vector of norms of machine usage costs PMT is given, the vector of norms of wages earned by various categories of employees PLT is also provided.

The value of external resources utilization (PER) E, the value of machine time usage (PMT) M, the value of labor time usage (PLT) L are calculated.

According to the terminology established in operations' research, the vectors of gross output $Z \ge O$, which satisfy the capacity limitations and provide for non-negative final product Y, are named feasible solutions or plans. Let us denote the set of plans by **Z**. Final product Y corresponds to each plan $Z \in \mathbf{Z}$. Let us denote the set of all possible final products by **Y**.

In geometry, the sets **Z**, **Y** are convex multi-dimensional polyhedrons or polytopes in Euclidean space $\mathbf{R}^{5,1}$. Broad research on convex polyhedrons exists, for example, in [18].

Let us assume that the vector of the product Y sales prices is 5-dimensional random number $P = (P_1\ P_2\ P_3\ P_4\ P_5)$, characterized by:

vector of expected values $E(P) := (E(P_1)\ E(P_2)\ E(P_3)\ E(P_4)\ E(P_5)) \in \mathbf{R}^5$,
vector of coefficients of variation $VC(P) := (VC(P_1)\ VC(P_2)\ VC(P_3)\ VC(P_4)\ VC(P_5)) \in \mathbf{R}^5$,
matrix of correlation coefficients $cor(P_1\ P_2\ P_3\ P_4\ P_5) := cor(P_i, P_j) \in \mathbf{R}^{5,5}$.

Stochastic revenue from sale of the final product equals TR = P Y.

Let us denote the stochastic profit by Π(Y). According to Bernoulli (μ, σ)-paradigm we are interested in

$E(TR) = E(P)\ Y$, $D(TR) = cov(P_1\ P_2\ P_3\ P_4\ P_5)\ Y \cdot Y$; $SD(TR) = D(TR)^{0,5}$.

Non-linear dual optimization problems

$max\ \{E(\Pi(Y)) \mid Y \in \mathbf{Y}, SD(\Pi(Y)) = const\}$, $min\ \{SD(\Pi(Y)) \mid Y \in \mathbf{Y}, E(\Pi(Y)) = const\}$

allow the manager to choose the level of output $Y \in \mathbf{Y}$, which either maximizes the expected value of profit at a given level of risk or minimizes the risk at a given level of the expected profit. The basic idea of this research is to transfer the risk diversification methods of stochastic income securities to management of a multi-product firm profit risk.

6. Empirical research on profit risk management with help of virtual firm model

Let us construct a TPF model of a virtual firm by concretizing indicators.

Table 3

The direct input coefficients at the firm "Franky," in natural units

	z_1	z_2	z_3	z_4	z_5
A :=	0.02	0.05	0.08	0.06	0.00
	0.01	0.00	0.17	0.00	0.02
	0.15	0.08	0.00	0.00	0.03
	0.00	0.05	0.04	0.07	0.12
	0.12	0.10	0.03	0.00	0.03
ER :=	0.15	0.00	0.40	0.30	0.15
	0.00	0.15	0.30	0.20	0.12
	0.10	0.05	0.25	0.10	0.20
MT :=	0.04	0.05	0.20	0.09	0.00
	0.08	0.02	0.06	0.10	0.05
LT :=	50.00	40.00	0.40	0.00	0.15
	6.00	3.00	40.00	90.00	0.20

It is provided that the production capacities are limited: $z_1 \leq 30$, $z_2 \leq 20$, $z_3 \leq 150$, $z_4 \leq 90$, $z_5 \leq 300$.

Below the prices for materials and services acquired externally, machinery depreciation rates, and wage rates of the firm "Franky" are provided. Initially, we regard all these prices as given – set by the respective markets. Mr. Frank is able to acquire the needed amounts of resources for constant prices:

$PER = (3\ 8\ 10)^T$, $PMT = (1.5\ 1.9)^T$, $PLT = (5\ 4)^T$.

The vectors of the product price expected values and coefficients of variation are given:

$E(P) = (300\ 200\ 150\ 90\ 70)$, $VC(P) = (0.01\ 0.08\ 0.02\ 0.15\ 0.10)$.

The matrix of the stochastic price vectors $P = (P_1\ P_2\ P_3\ P_4\ P_5)$ component correlation coefficients is provided (Table 4).

Table 4

The firm "Franky" product price correlation matrix

cor(P) =

1	0.1	-0.8	0	-0.5
0.1	1	0.2	-0.6	0.3
-0.8	0.2	1	0	-0.9
0	-0.6	0	1	0.4
-0.5	0.3	-0.9	0.4	1

In Figure 1, the Bernoulli set **B** of the firm "Franky" is provided.

Ordinate of the Bernoulli set upper limit, which corresponds to the abscissa σ is

$\mu^{max}(\sigma) := max\ \{E(\Pi(Y)) \mid Y \in \mathbf{Y}, SD(\Pi(Y)) = \sigma\}$ with Lagrange multiplier $\lambda^{max}(\sigma)$.

Ordinate of the Bernoulli set lower limit, which correspons to the abscissa σ is

$\mu^{min}(\sigma) := min\ \{E(\Pi(Y)) \mid Y \in \mathbf{Y}, SD(\Pi(Y)) = \sigma\}$ with Lagrange multiplier $\lambda^{min}(\sigma)$.

The upper frontier of the Bernoulli set **B** in range of risk $\sigma \in [0; 1591.59]$ is the Pareto frontier **P**. Let us note, that by assuming increased risk within a range $\sigma \in [1591.59; 2400]$, the maximum expected profit decreases. Thus, experiments with a virtual firm let us to conclude that increased risk not always leads to an increase in the expected profit. In their explanation of entrepreneurial profit, economists widely build on the theses expressed by Joseph Alois Shumpeter [9] and Frank Knight [19] that the producer earns abnormal profit, if he assumes risk. In our research, numerical experiments with a virtual firm make this thesis more precise. Increased risk itself does not warrant increase in the expected profit. First, as it is depicted in Figure 2, if too high a level of risk is chosen, the maximum expected profit begins to decrease. Second, an appropriately chosen level of risk generates abnormal profit for an entrepreneur under the condition that the entrepreneur was able to maximize the expected profit that corresponds to the assumed level of risk. As other experiments demonstrate, if the entrepreneur chooses a low risk strategy, the maximum expected profit could be negative. Concerning this statement, we define a concept of risk breakeven. Risk breakeven is that minimum level of risk, starting from which the maximum expected profit becomes positive.

The calculations and Figure 1 illustrate the behavior of Lagrange multipliers.

For example, under a condition that $SD(E(\Pi)) = 700$ the maximum expected profit equals 23394.75, and Lagrange multiplier is $\lambda^{max}(700) = 0.004394$, and the minimum expected profit equals −1531.88 with Lagrange multiplier $\lambda^{min}(700) = -0.001430$.

Under a condition that $SD(E(\Pi)) = 2000$ the maximum expected profit equals 25959.02 and Lagrange multiplier is $\lambda^{max}(2000) = -0.001042$, and the minimum expected profit equals 8727.23 with Lagrange multiplier $\lambda^{min}(2000) = 0.002356$.

As it has been stated, the Lagrange multipliers are interpreted as the marginal maximal (or minimal) expected profit with respect to risk. It is interesting to mark the empirically obtained support to the law of diminishing the marginal indicator $\lambda^{max}(\sigma)$, as it was expected, even though Figure 1 is misleading in this respect.

For example, $\lambda^{max}(100) = 0.013705$, $\lambda^{max}(200) = 0.011024$, $\lambda^{max}(300) = 0.008747$, $\lambda^{max}(400) = 0.007126$, $\lambda^{max}(700) = 0.004394$, $\lambda^{max}(1500) = 0.001395$, $\lambda^{max}(1600) = -0.001837$.

At a chosen level of risk $\sigma = 1600$ the maximum expected profit is at maxium and equals 27874.

If $\sigma > 1600$, then the maximum expected profit decreases. Thus, concern for maximal efficient risk is well-founded.

In Figure 2, a function $\mu^{max}(\sigma)$ that relates the maximum expected profit to the level of risk is depicted.

Three sigma area $[\mu^{max}(\sigma) - 3\ \sigma; \mu^{max}(\sigma) + 3\ \sigma]$ of the stochastic profit $\Pi(\sigma)$, where the stochastic profit is concretized with a probability larger than 0.88, is also depicted.

We have reviewed the theoretical model with general indicators μ_i, σ_i, $\rho_{i\,j}$. In practice, the observed empirical estimators of general indicators that have been calculated with the help of the statistical data (sample) are applied in management of the product price volatility risk. The empirical estimation errors of general population indicators that have been applied in risk diversification are studied in the paper [20].

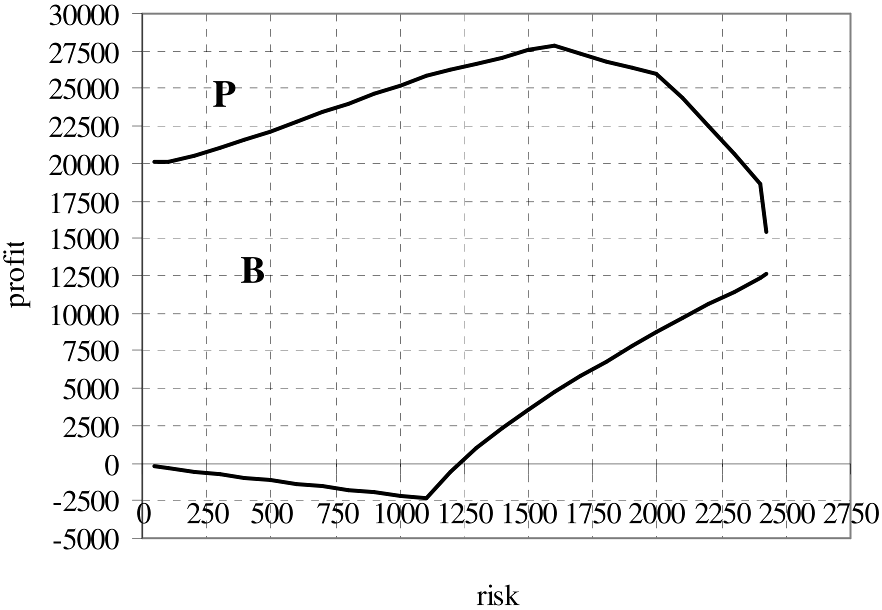

Fig. 1. **Firm's "Franky" Bernoulli set B. The upper frontier of Bernoulli set B in the risk area $\sigma \in [0; 1591.59]$ is Pareto frontier P. Maximal efficient risk equals 1600**

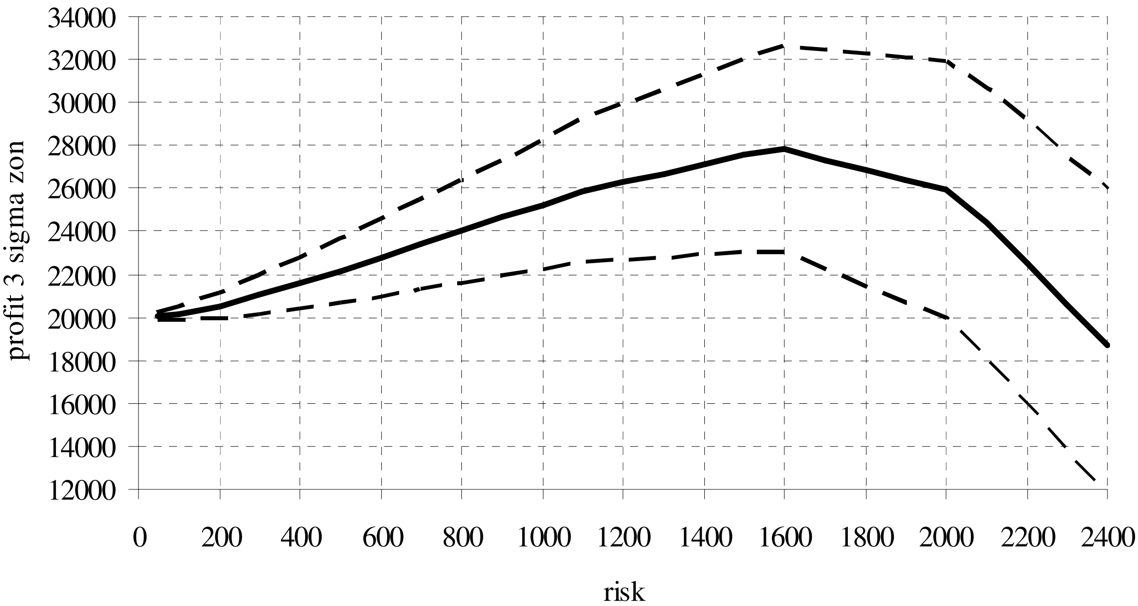

Fig. 2. **Graph of maximum expected profit as a function of risk and corresponding "minus three sigma" and "plus three sigma" lines**

Conclusions

1. The experiments with a virtual firm ascertain that, if a high risk strategy is chosen, the maximum expected profit could decrease, therefore, consideration of maximal efficient risk is well-founded.
2. The calculations involving the data from the database "Consumer prices" (The CSB of the Republic of Latvia) attest that diversification of risks presented in this paper is applicable in practice of agricultural production in Latvia.
3. In the calculations of risk diversification, a model with a perfectly structured technology may be replaced by a more general firm's TPF model.

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