CALCULATION OF DEFORMATION OF RUBBER LAYER IN RUBBER METAL ELEMENTS

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Abstract. In solid mechanics, there are classical methods for determining and calculating the deformation of the material body, which is used today. Such material, like rubber is anisotropic and at large deformations classical equations and relations are not applicable. For calculation of large deformations notion nonlinearity exists. The given paper contains the use of a non-traditional method for calculating the stress-strain state of rubber metal supports. This method is based on the Cauchy tensor to determine deformation of the body when there is displacement of the body points. With the help of Matlab system we wanted to show the applicability of the new method for calculation of the body with large deformations.

Keywords: rubber, large deformation, Cauchy tensor, displacement, compression.

Introduction

Let us write the universally recognized statement of static boundaries values problem as given in the reference [1]. Body with predetermined forces within its volume *V* and on the surface *S* is in the equilibrium state. We find the stresses and strains within of the investigated material body. The volume of the investigated body *V* and the surface *S*, should be knew beforehand (we need to know the geometrical parameters of the investigated body), if these parameters are not known, it means the external forces are not specified, too.

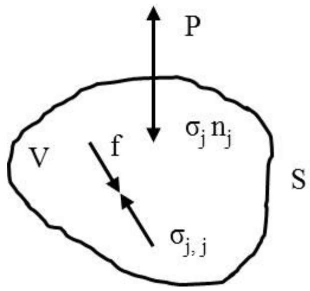

Fig. 1. **At any point within the volume *V* of the body on the surface *S* and the external forces balanced by internal stresses:** the stress vector σ_j at the surface with the normal vector x_j

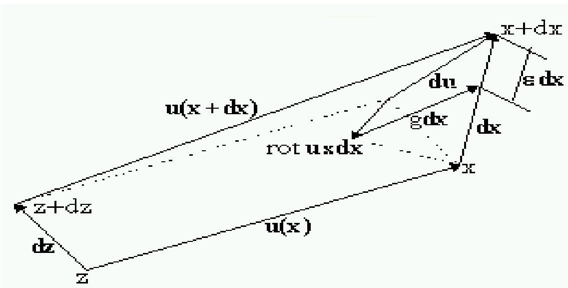

Fig. 2. **Decomposition of the vector relative displacement**

Assume that f_i and p_i are external forces set in the volume *V* and on the surface *S*. Denoting by stress components σ_{ij} is representing the statement mathematically.

$$\sigma_{ji,j} + f_i = 0, \quad \sigma_{ij} = \sigma_{ji}, \quad x_i \in V , \tag{1}$$

$$\sigma_{ij,kk} + \frac{1}{1+v}\sigma_{kk,ij} + \frac{v}{1-v}\delta_{ij} f_{k,k} + f_{i,j} + f_{j,i} = 0, \; x_i \in V , \tag{2}$$

$$\sigma_{ji} n_j = p_i, \quad x_i \in S , \tag{3}$$

where v – Poisson's ratio.

The vector of the relative displacement can be represented as

$$du_i = u_{i,j} dx_j = \left(\varepsilon_{ij} + \omega_{ij}\right) dx_j \tag{4}$$

where $\varepsilon_{ij} = \left(u_{i,j} + u_{j,i}\right)/2, \quad \omega_{ij} = \left(u_{i,j} - u_{j,i}\right)/2$

The projection of the relative displacement on the direction of the vector dx_i

$$du_i\, n_i = \varepsilon_{ij}\, n_i\, dx_j = \varepsilon\, dx\ ,$$

where η – direction cosines of the vector dx_i,

$$\varepsilon = \varepsilon_{ij}\, n_i\, n_j,\ \ dx = \left(dx_i dx_i\right)^{1/2}. \tag{5}$$

Further for brevity $\left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right)$ will be denoted by the matrix

$$g = \left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right)$$

In expanded form, this matrix has the form

$$g = \left(g_{ij}\right) = \begin{bmatrix} \varepsilon_{11} - \varepsilon & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} - \varepsilon & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} - \varepsilon \end{bmatrix}.$$

Assume that the right side of the equation (4) is equal to zero

$$\varepsilon dx_i - \varepsilon\delta_{ij} dx_j$$

and we will write it in the form

$$du_i = \varepsilon dx_i + \left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right) dx_j + \omega_{ij} dx_j\ . \tag{6}$$

Expression (6) decomposes the vector of the relative displacement of the vectors:

- elongation of the vector direction $dx_i - \varepsilon dx_i$,
- shear – $\left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right) dx_j$
- rotation – $\omega_{ij} dx_j$

It is not difficult to verify that the vector of shear and rotation is perpendicular to the vector dx_i. The scalar multiplication of the vectors, mentioned above by the vector dx_i.

$$\left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right) dx_j dx_i = \left(\varepsilon_{ij} - \varepsilon\delta_{ij}\right) n_j n_i dx^2 = \left(\varepsilon_{ij} n_j n_i - \varepsilon\delta_{ij} n_j n_i\right) dx^2 = \left(\varepsilon - \varepsilon\right) dx^2 = 0;$$

$$\omega_{ij} dx_j dx_i = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right) dx_j dx_i = \frac{1}{2}\left(u_{i,j} dx_j dx_i - u_{j,i} dx_j dx_i\right) = \frac{1}{2}\left(u_{j,i} dx_i dx_j - u_{i,j} dx_i dx_j\right) = 0.$$

The square of the length of the shear vector

$$\left(\varepsilon_{ki} - \varepsilon\delta_{ki}\right)\left(\varepsilon_{kj} - \varepsilon\delta_{kj}\right) dx_i dx_j = \left(\varepsilon_{ki} - \varepsilon\delta_{ki}\right)\left(\varepsilon_{kj} - \varepsilon\delta_{kj}\right) n_i n_j dx^2 = \left(\varepsilon_{ki}\varepsilon_{kj} n_i n_j - \varepsilon^2\right) dx^2 = \gamma^2 dx^2$$

where $\gamma = \left(\varepsilon_{ki}\varepsilon_{kj} n_i n_j - \varepsilon^2\right)^{1/2}$ – relative shear deformation.

From Fig. 2, let us expand the vector of relative displacement

$$dz_i = dx_i - du_i\ . \tag{7}$$

If we are taking to square both sides of the expression (7), the result is represented in the form

$$dx^2 - dz^2 = \left(2\varepsilon_{ij} - u_{k,i} u_{k,j}\right) dx_i dx_j = 2a_{ij} dx_i dx_j\ , \tag{8}$$

where

$$a_{ij} = \varepsilon_{ij} - \frac{1}{2} u_{k,i} u_{k,j} .$$

Expression (8) is introduced into the theory of elasticity and other sections of solid mechanics by the following view: if multiplication $u_{k,i}$, $u_{k,j}$ is negligible in comparison with $u_{k,i}$, the deformed condition can be characterized as ε_{ij}. Otherwise, strain should be submitted to a_{ij}. In accordance with this, ε_{ij} becomes known as the tensor of small and infinitesimal deformations, a_{ij} – tensor of finite deformations. In addition, equation (8) in solid mechanics is allocated a fundamental place.

Nontraditional methods of solution of static boundary value problem

Assume that the solution of the static boundary problem is known, in this work it will be called "non-traditional solutions". From it, deformation is easily determined

$$\varepsilon_{ij} = \frac{1}{E}\left(-v \cdot \delta_{ij} \cdot \sigma_{kk} + (1+v)\sigma_{ij}\right), \tag{10}$$

where E – modulus of elasticity.

The displacement is defined by using Cesaro formulas [2].

$$u_i(x) = u_i(x^0) + \omega_{ij}(x^0)(x_j - x_j^0) + \frac{1}{E}\int_l \left(\varepsilon_{ik}(y) + (x_j - y_i)\left(\varepsilon_{ki,j}(y) - \varepsilon_{kj,i}(y)\right)\right) dy_k$$

where l – line in the area of V;
x^0 – initial point of this line;
$u_i(x^0)$, $\omega_{ij}(x^0)$ – are integration constants. It is more convenient to use transformed forms of it [3].

$$u_i(x) = u_i(x^0) + \omega_{ij}(x^0)(x_j - x_j^0) +$$

$$+\frac{1}{E}\int_l \left(-v\delta_{ik}\sigma_{tt} + (1+v)\left(\sigma_{ik} + (x_j - y_j)\left(-v\left(\delta_{ik}\sigma_{tt,j} - \delta_{kj}\sigma_{tt,i}\right) + (1+v)\left(\sigma_{ki,j} - \sigma_{kj,i}\right)\right)\right)\right) dy_k$$

In this expression $u_i(x^0)$, $\omega_{ij}(x^0)$ – are arbitrary constants. They correspond to displacement, which originates without deformations (parallel displacement and rigid rotation of solid body). Subsequently, it will be excluded from consideration of such displacement. In this case

$$u_i(x) = \frac{1}{E}\int_l \left(-v\delta_{ik}\sigma_{tt} + (1+v)\left(\sigma_{ik} + (x_j - y_j)\left(-v\left(\delta_{ik}\sigma_{tt,j} - \delta_{kj}\sigma_{tt,i}\right) + (1+v)\left(\sigma_{ki,j} - \sigma_{kj,i}\right)\right)\right)\right) dy_k \tag{11}$$

The calculated part by non traditional methods in Matlab

Let us demonstrate the correctness of the nominated in this paper new provisions by solving the task [1].

The domain of definition will be presented by equations of the static boundary problem in the form specified in Fig. 2, the rubber-metal element with one rubber layer. Initial coordinates are located in the center, which corresponds to the position (X, Y, Z) = (0, 0, 0).

By using the symbol V the volume of the following area will be denoted

$$-5 \le x_1 \le 5, \quad -5 \le x_2 \le 5, \quad 0 \le x_3 \le h, \tag{12}$$

The second boundary problem without mass forces will be considered.

$$\sigma_{ji,j} = 0, \quad \sigma_{ij} = \sigma_{ji}, \quad x_i \in V, \tag{13}$$

$$\sigma_{ij,kk} + \frac{1}{1+v}\sigma_{kk,ij} = 0, \ x_i \in V, \tag{14}$$

$$\sigma_{ji} n_j = \delta_{i2}\, c\, x_3, \quad x_i \in S, \tag{15}$$

where V – determined by the equations (12).

The problem (13-15) is completely determined mathematically. It has a simple mechanical meaning – the rubber-metal support with the efforts (15) on the upper plates stand in the state of equilibrium. It is required to find the stress-strain state of every inner point of this rubber-metal support. As can be seen, there is no deviation from the standard formulation of the static boundary problem.

Non-traditional solution of the problem

$$\sigma_{ij} = \delta_{i2}\delta_{j2}\, c\, x_3, \quad x_i \in V, \tag{16}$$

The functions of displacements are possible to define if the expression (16) will be put into equation (11):

$$u_i = \frac{1}{E}\int_l c\Big(-v\delta_{ik}x_3 + (1+v)\delta_{i2}\delta_{k2}x_3 + (x_j - y_j)\Big(-v\big(\delta_{ki}\delta_{3j} - \delta_{kj}\delta_{3i}\big) + (1+v)\delta_{k2}\big(\delta_{i2}\delta_{3j} - \delta_{j2}\delta_{3i}\big)\Big)\Big)dy_k,\ x_i \in V.$$

When integrated this expression, it will be obtained

$$u_i = -c\Big(\delta_{i1}vx_3\big(x_1 - x_1^0\big) - \delta_{i2}x_3\big(x_2 - x_2^0\big) + \\ + \delta_{i3}\Big(x_2^2 + v\big(x_3^2 - x_1^2\big) - x_2^0\big(2x_2 - x_2^0\big) - v\Big(\big(x_3^0\big)^2 - x_1^0\big(2x_1 - x_1^0\big)\Big)\Big)/2\Big)/E,\ x_i \in V, \tag{17}$$

where x_i^0 – any fixed point of the area V.

Below are given detailed views of the functions (17):

$$u_1(x) = -\frac{c\, v\, x_3\big(x_1 - x_1^0\big)}{E},\ x_i \in V \quad and \quad u_2(x) = \frac{c\, x_3\big(x_2 - x_2^0\big)}{E},\ x_i \in V,$$

$$u_3(x) = -c\Big(\Big(x_2^2 + v\big(x_3^2 + x_1^2\big) - x_2^0\big(2x_2 - x_2^0\big) - v\Big(\big(x_3^0\big)^2 - x_1^0\big(2x_1 - x_1^0\big)\Big)\Big)/2E\Big),\ x_i \in V.$$

The functions (17) satisfy the differential equilibrium equations of Navier's form [4]. From the displacement fields (17) the components of deformation and rotation are defined

$$\varepsilon_{ij} = cx_3\Big(-v\big(\delta_{i1}\delta_{j1} + \delta_{i3}\delta_{j3}\big) + \delta_{i2}\delta_{j2}\Big)/E,\ x_i \in V, \tag{18}$$

$$\omega_{ij} = -c\Big(v\big(x_1 - x_1^0\big)\big(\delta_{1i}\delta_{3j} - \delta_{3i}\delta_{1j}\big) - \big(x_2 - x_2^0\big)\big(\delta_{2i}\delta_{3j} - \delta_{3i}\delta_{2j}\big)\Big)/E,\ x_i \in V. \tag{19}$$

According to the obtained expressions at any point in the area V the body is in equilibrium, it is possible to define the stress components, deformation and rotation. But especially we will note that in all expressions (16-19) the coordinates of the area V were used.

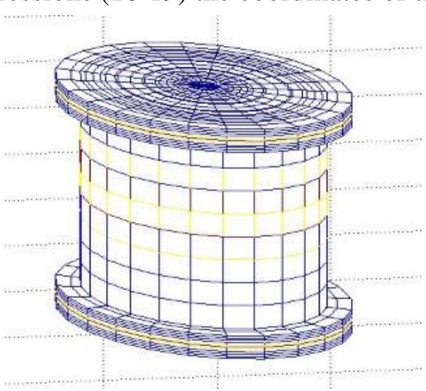

Fig. 2. **Rubber metal supports in the state of equilibrium**

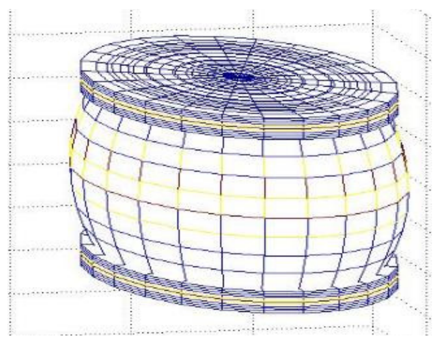

Fig. 3. **Compression of rubber-metal supports under a uniformly distributed load**

In the final state, the studied body has the same configuration and occupies the same position in space. This body has an immovable and geometrically unchangeable position under any values of external load.

Figure 2-5 are obtained by using the Matlab system. In this system, all expressions are recorded via a special code, which is given in a summarized form in [5]. According to the obtained equations displacement of points of the rubber-metal support when exposed to compressive force are built.

After calculation in Matlab we can get the value of displacement of each point of the rubber layer. In the initial condition at formulation of the problem in Matlab it was divided by 25 points of the rubber layer. Table 1 gives the coordinates of each of the first, 3rd, 5th, etc. points.

Table 1

The calculation in Matlab the displacement points of the rubber layer

	axes	The coordinates of the points of the rubber layer, mm												
0 MPa	**Z**	0	8	17	25	33	42	50	58	67	75	83	92	100
	X, Y	120	120	120	120	120	120	120	120	120	120	120	120	120
	axes	The coordinates of the points displacement of the rubber layer, mm												
4 MPa	**Z**	0	8	16	24	32	40	48	56	64	72	80	88	96
	X, Y	120	121.5	122.8	123.7	124.4	124.9	125	124.9	124.4	123.7	122.8	121.5	120
8 MPa	**Z**	0	6	13	19	26	32	38	45	51	58	64	70	77
	X, Y	120	126.1	131.1	135	137.8	139.4	140	139.4	137.8	135	131.1	126.1	120

At the given points in the table, displacement of the same points in the plane for a visual presentation was constructed. In the same figure displacement for different values of *a* is compatible, which will monitor the process of points moving during compression. The figure shows the coordinate values of each point of the body during movement towards the vertical axis and the horizontal axis (illustrated buckling of the rubber layer at the edges).

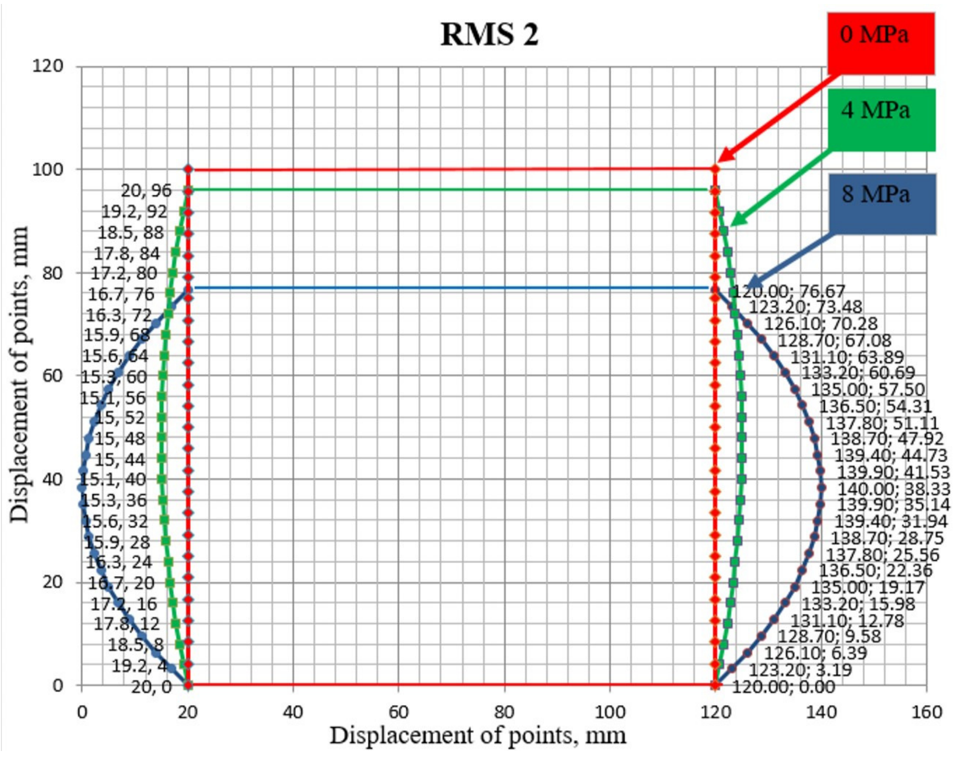

Fig. 4. **Displacement of contour points of the researched rubber body of RMS2**

To determine the impact on the bearing capacity and natural frequency two types of rubber metal supports were made. Structurally RMS2 looks like as follows, with two end metal plates with thickness – 4 mm, and the rubber layer has a thickness – 92 mm. RMS3 has two rubber layers with thickness 44 mm, structurally, it looks like the RMS2, but includes three metal plates and one of them in the middle is divided by the rubber layers in two equal parts [5].

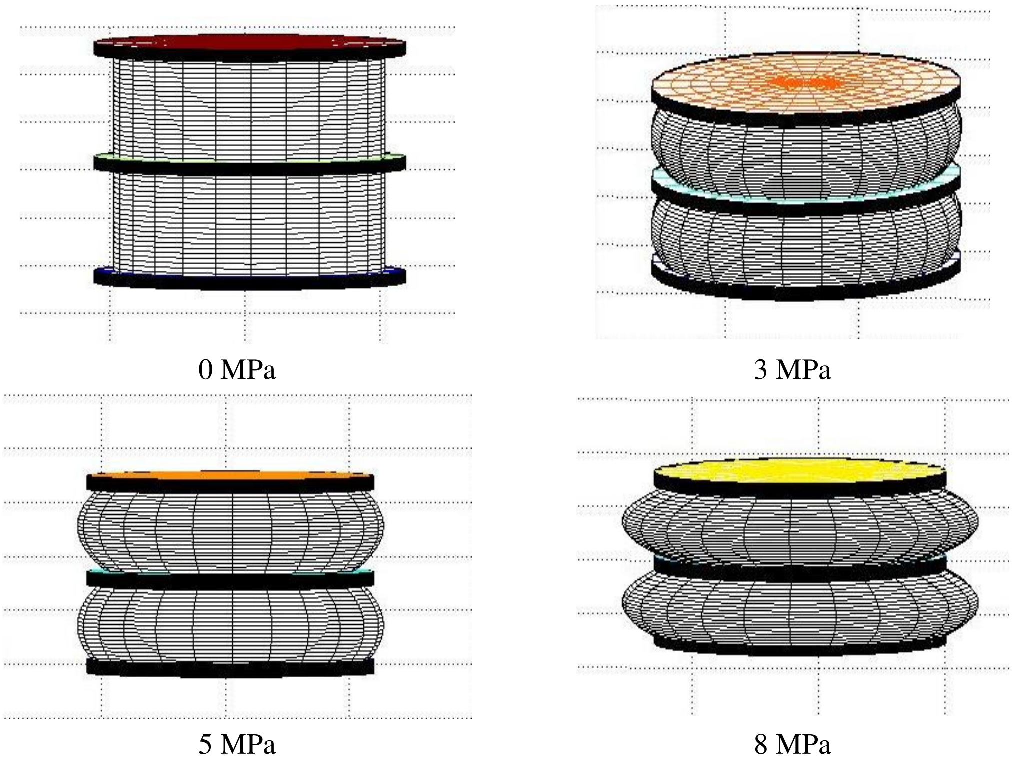

Fig. 5. **Process of deformation RMS3 for various compression forces, the distributed load applied to the upper end**

The calculated part in the ANSYS system with the use of the finite element method

Features of ANSYS software, static structural analysis are used to determine displacements, stresses, deformations that arise in the construction or its component parts by application of mechanical forces. Static analysis is suitable for problems, in which the action of the forces of inertia or of energy dissipation processes does not significantly affect the behavior of the structure. This type of analysis is possible to be used in many applications, for example, to determine the stress concentration in hollow articles or of structural components for calculation of temperature stresses.

Engineers and specialists in the field of strength are familiar with this kind of analysis, and probably they solved many problems of static or classical methods by using the ratio of the respective directories. In ANSYS numerical methods are used to solve these problems. The resolving equations of static analysis are written as

$$[K]\{u\} = \{F\}, \tag{21}$$

where $[K]$ – stiffness matrix;

$\{u\}$ – displacement vector.

Components of the forces vector $\{F\}$ may be concentrated forces, thermal load, pressure and force of inertia. It is possible to perform calculations by definition of the values of acceleration, which provide a static balancing of loads applied to the system.

Static analysis in the ANSYS program can include non-linearity as plasticity and the creep of the material, large deflections, large deformations and contact interaction. By nonlinear static analysis typically performed by gradually increasing the loads it is possible to receive a correct decision. [6]

Solution of the given the boundary problem by the finite element method in ANSYS program was carried out in three phases according with the logic of the method.

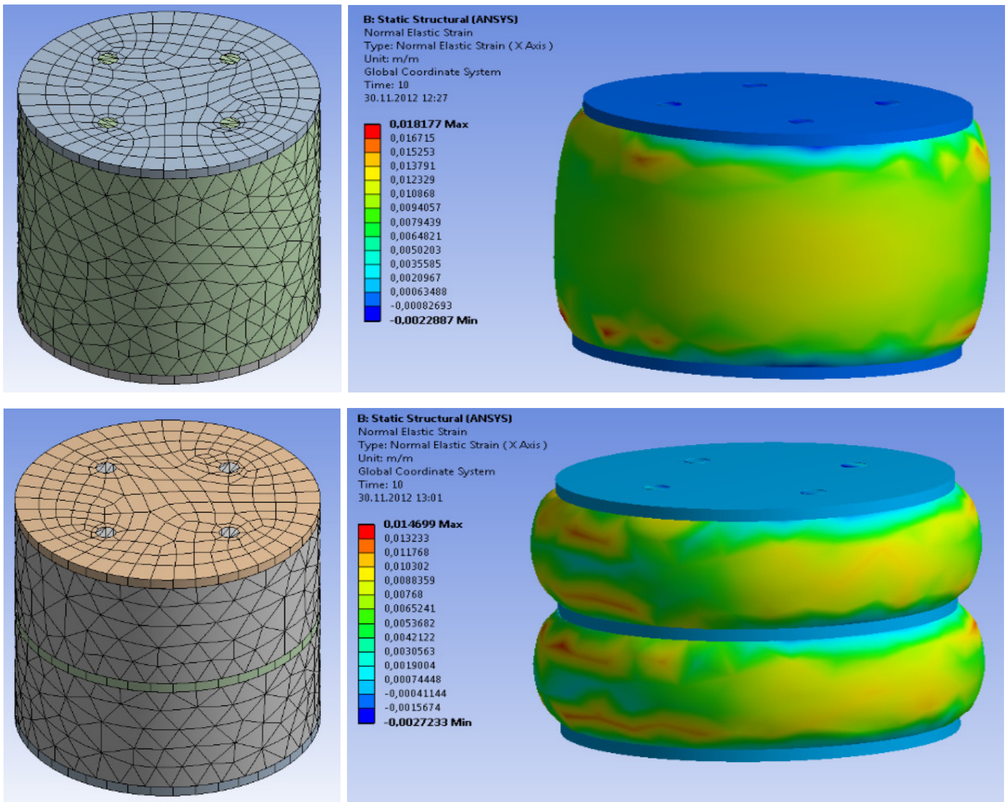

Fig. 7. **Modelling of compression RMS2, RMS3 under compression by using the finite element method in ANSYS**

Experimental part

Conducting the experiment to determine the physical and mechanical properties of rubber in metal elements is very important. The behavior of the rubber layer often depends on the brand of rubber and its manufacturing technology, the geometry of the rubber component, service conditions; knowing exactly that to predict mathematically the behavior of the rubber layer under the influence of dynamic loads is not possible. [7] The samples have the following geometry: height 105 mm and diameter 100 mm. The plates of these samples were made of steel (St.3) and the rubber compound TU.38.105.1082-86. SNK Mark MBS - 3826, butadiene – nitrile.

When designing the RMS it is necessary to uniform distribution of stress in the rubber layer as given in [8], and to exclude the stress concentration. The surface of the metal plate in such an element must be well processed for the exception of on its corners, protrusions, cavities or holes. Because this construction is designed to perceptions by long-term large loads under compression, it is an expedient to manufacture such constructions with preliminary compression of the rubber elements or without tensile stresses.

Characteristics of stiffness of rubber-metal supports can change both due to the use of the brand of rubber and by constructional features (geometry of the rubber and the whole structure). Stiffness can be changing (and therefore, the natural frequency) in different directions and may be carried out by changing the size or the rubber elements or by an additional metal plate. To conduct the compression test the press machine UM 100 was used. During compression RMS bulge occurs at the edges of the rubber layer. Metal plates practically are not deformed and their function is only to distribute the load uniformly on the rubber layer [9]. When unloading the rubber layer it is completely restored and gets the original appearance. During buckling of the rubber layer external damage was not observed.

Fig. 6. **Compression test of RMS2 and RMS3**

Comparison of calculations and experimental data

According to the results of the experiments the curve of dependence of the stress-strain state of rubber-metal supports is based.

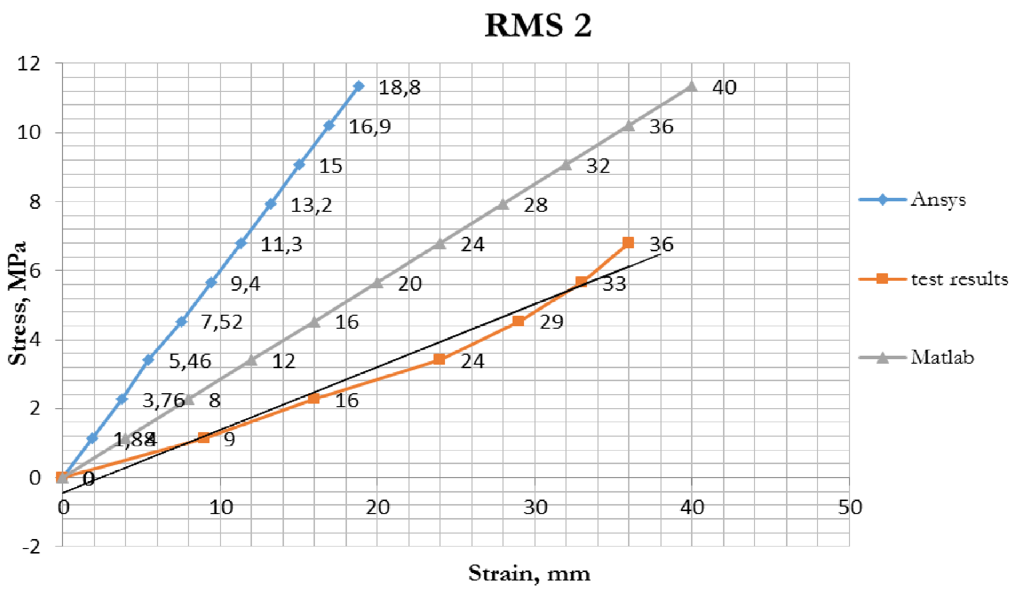

Fig. 8. **Stress-strain state of rubber-metal support**

According to the calculations in the Matlab system we built the graph of displacement of the points in the rubber-metal supports. 24 points of the surface (contour points of body) have been taken. Figure 5 shows the difference between the calculated and the experimental data. The calculated results from Matlab and ANSYS are also distinguished. Using of the new non-traditional method is suitable for describing the deformation of rubber-metal elements in the field of small and finite deformation state of the rubber layer.

Conclusions

The proposed new method allows us to determine the stress-strain state of the studied body under loading by using the analytic dependence. It is possible when the Cauchy tensor for determination of displacement of all points of the body is used. The obtained results from the conducted experiments and analysis are quite helpful for engineers and designers designing active seismic vibration protection of buildings and other engineering structures. The conducted research reveals applicability of the new non-traditional method for calculations based on analytic expressions; it is applicable like other methods.

Acknowledgement

R. Askarbekov thanks to the CASIA project http://www.eu-casia.org/home, it gave a possibility to visit the Czech University of Life Sciences Prague and finish this article.

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