MODELLING OF HEAT TRANSFER PROBLEM IN LAYERED GYPSUM WALL WITH DIFFERENT GYPSUM DENSITIES IN CASE OF FIRE

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Abstract In this paper we study the problem of the heat transfer through gypsum board products exposed to fire. The paper proposes a thermal conductivity model in the gypsum product layer for gypsum plasterboard and foam gypsum with different density ρ and at high temperature T. For a nonlinear problem the specific heat cp and thermal conductivity K coefficients depend on temperature. For analysing the heat transfer, the non-stationary partial differential equation (PDE), expressing the rate of change of temperature T in every layer is considered. The following problems are studied: heat transfer through one ($N = 1$), two ($N = 2$) and three ($N = 3$) different densities layered material of gypsum exposed to fire. The system of N non-stationary nonlinear partial differential equations (PDEs), expressing the rate of change of temperature T in each layer is considered. The approximation of the corresponding initial boundary value problem of this system is based on the finite difference method by using the MATLAB routine “pdepe”. The results of heat transfer calculations for 2 and 3 layers have been compared with the experimental data obtained at the Latvia University of Life Sciences and Technologies.

Keywords: gypsum board, foam gypsum, PDEs, fire resistance.

1. Introduction

Gypsum boards are widely used for wall and floor structures, but little information is given for the failure times of gypsum-based boards in fire conditions, in EN 1995-1-2 or in standards for gypsum-based boards. The failure time of the board is an important property for the fire safety design of timber frame constructions.

Gypsum contains chemically bound water of crystallization. The fire safety properties of gypsum boards are mainly due to this water content. When the gypsum board is exposed to fire, the water of crystallization gradually separates and evaporates, consuming a large amount of energy in the process and delaying the transfer of heat through the board. The gypsum board effectively acts as a fire barrier until most of its water is removed. During a fire, the effect of gypsum panels is a typical isothermal stage on the unexposed side due to the latent heat effect. This dehydration causes cracks inside the panels. The heat flow passes through these cracks and raises the temperature of the unexposed side. Dehydration causes high thermal shrinkage due to the associated loss of water inside the plate.

The decay times of gypsum plasterboards in wooden walls and floors, from full-scale fire tests are shown [1; 2]. It is indicated there that the decay times of gypsum boards under the floors are shorter than under the walls due to gravity.

The development of new gypsum-based fire protection materials (which have high temperature resistance) must reduce high temperature shrinkage. Various solutions are offered to reduce heat transfer, for example, silica filler is applied to gypsum plaster [3]. An important issue is the behavior of gypsum boards subjected to mechanical stress and thermal exposure [4]. It has been found experimentally that the formulas developed to calculate the bending braking load and the failure time in the case of combined operation need to be clarified.

The volume weight of widely used gypsum boards is 600-1000 kg·m^{-3}, but foam gypsum can be obtained with a volume weight of 250-350 kg m^{-3}. This makes it possible to include in the composition of foam gypsum the fire resistance and beneficial properties of porous materials. The thermal properties of gypsum boards in the heating process have been studied. Temperature and heating rate-dependent properties of gypsum board material, such as λ, cp and ρ, were determined experimentally [5; 6]. A lighter version of gypsum has been developed in Latvia - foam gypsum. The first experimental and theoretical studies at high temperatures to determine the fire resistance of foam gypsum were performed [7-9].

The both gypsum products can be used together or separately in wall and floor structures. An important issue is the protection of fire of such walls. In order to find out the fire protection of different

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uses of gypsum boards and foam gypsum, it is necessary to compile a heat transfer model for each of the situations.

The problem of the heat transfer through N – layered materials of gypsum board products exposed to fire is studied. The hybrid experimental-numerical method of solving heat and mass transfer through different media taking into account dependency on temperature with some of the coefficients' (c_p (specific heat) and K (thermal conductivity)) is studied in papers [10; 11].

The temperature dependency of the coefficients was modelled with interpolation formulas and applied in the given paper to investigate non-linear boundary problems. Being based on the papers the temperature dependency of the coefficients c_p and K was modelled with interpolation formulas and applied in the given paper to investigate non-linear boundary problems [12; 13].

The paper proposes and analyses the temperature field calculation methodology for layered materials in a high temperature heating process.

2. Materials and methods

For modelling we use a foam gypsum plate with density $\rho = 300$ kg·m^{-3} and gypsum boards with density $\rho = 1000$ kg·m^{-3}. The temperature function $g(t) = 20 + 345 \cdot \lg(8t + 1)$ ºC is used to model the fire, where t is the time in minutes. These values are chosen because they are close to the densities of the foam gypsum and gypsum board used in the experiment. Three different situations are considered:

1. Gypsum board material with one layer plate in x-direction: a foam gypsum plate with thickness x = 0.025 m and a similar thickness gypsum board (2 connected gypsum boards). There one side of the plate's layer is heated (fired) with the temperature function $g(t)$.
2. The heat transfer processes in two layers ($N = 2$) of gypsum material, with different densities, at high temperatures are studied. The thickness of each separate material layer is 0.025 m, i.e. the total thickness of the material is 0.05 m. The heat transfer of this layer is modelled, assuming that the fire can be on the foam gypsum or gypsum board sides.
3. A layer consisting of three layers of gypsum material of different densities is considered. The model considers the case when a foam gypsum plate with a thickness of 0.04 m is boundary by gypsum boards with a thickness of 0.0065 m, i.e. the total layer thickness is 0.053 m (it is often used to create partitions).

The experimental measurements data in the burning-heating process of the foam gypsum and gypsum boards is done at the Latvian University of the Life Sciences and Technologies (Faculty of Environment and Civil Engineering). The operation of the experimental equipment can be seen in Fig. 12 and Fig. 16.

3. The mathematical model

The process of diffusion and heat transfer is considered in 1-D space domain

$$\Omega = \{(x, y, z) : 0 \le x \le L, -\infty \le y \le \infty, -\infty \le z \le \infty\}.$$

The domain Ω consists of different N-layer medium.

We will consider the non-stationary problem of the nonlinear diffusion theory for N- layered piece-wise homogenous materials in the domain

$$\Omega_i = \{(x, y, z) : x \in (x_{i-1}, x_i), y \in (-\infty, \infty), z \in (-\infty, \infty)\}, i = \overline{1, N},$$

where $H_i = x_i - x_{i-1}$ is the height of the layer Ω_i;
$x_0 = 0$;
$x_N = L$.

The rate at which the temperature of the material changes in N layers is determined by the heat conduction PDE in the following form:

$$c_{pi}\rho_i \frac{\partial T_i}{\partial t} = K_i \frac{\partial^2 T_i}{\partial x^2}, x \in [x_{i-1}, x_i], i = \overline{1, N}, t > 0, \tag{1}$$

where c_{pi} – specific heat;

K_i, ρ_i – heat conductivity and the density of the gypsum material.

We assume that the coefficients c_{pi} and K_i in the PDEs are dependent on the temperature T similarly as in [6]. The aforementioned dependence on temperature is obtained due to the spline interpolation, which is shown in [14].

3.1. One layer case

We will consider the non-stationary problem of the linear diffusion theory for layered homogenous materials in this domain ($N = 1$).

In the layer we get the nonlinear PDEs

$$a^2(T)\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t}, \tag{2}$$

where $a^2(T) = \dfrac{K(T)}{\rho c_p(T)}, i = 1, 2$ – thermal diffusion coefficients depending on T.

For the initial condition for $t = 0$ we give $T(x,0) = T_0$, where $T_0 = 20$ ºC.

We use the following boundary conditions:

$$\frac{\partial T(0,t)}{\partial x} = 0, T(L,t) = g(t). \tag{3}$$

3.2. Two layer case

We will consider the non-stationary problem of the nonlinear diffusion theory for two layered piece-wise homogenous materials in the domain Ω_i, $i = \overline{1,2}$, where $x_1 = H_1 = 0.025$ m, $x_2 = L = H_1 + H_2 = = 0.05$ m, $H_2 = 0.025$ m.

In two layers ($N = 2$) we get the system (1) of two PDEs

$$\begin{cases} D_1(T)\dfrac{\partial^2 T_1(x,t)}{\partial x^2} = \dfrac{\partial T_1(x,t)}{\partial t}, \\ D_2(T)\dfrac{\partial^2 T_2(x,t)}{\partial x^2} = \dfrac{\partial T_2(x,t)}{\partial t}, \end{cases} \tag{4}$$

where $D_i(T) = \dfrac{K_i(T)}{\rho_i c_{pi}(T)}, i = 1, 2$ – thermal diffusion coefficients depending on T.

For the initial condition for $t = 0$ we give $T_1(x,0) = T_2(x,0) = T_0$, where $T_0 = 20$ ºC.

We use the following boundary and continuous conditions:

$$\begin{cases} \dfrac{\partial T_1(0,t)}{\partial x} = 0, T_2(L,t) = g(t), \\ T_1(x_1,t) = T_2(x_1,t), D_1(T)\dfrac{\partial T_1(x_1,t)}{\partial x} = D_2(T)\dfrac{\partial T_2(x_1,t)}{\partial x}, \end{cases} \tag{5}$$

where $g(t) = 20 + 345 \cdot \lg(8t + 1)$ ºC,
t – time in minutes.

A much more detailed description of the theoretical solution of the given problem, the use of the solution method and the justification can be found [15].

3.3. The case of three layers

We will consider the non-stationary problem of the nonlinear diffusion theory for three-layered piece-wise homogenous materials in the domain Ω_i, $i = 1,\overline{2},3$ where $x_1 = H_1 = 0.0065$ m, $x_2 = H_1 + H_2$ m, $H_2 = 0.040$ m, $x_3 = L = 2H_1 + H_2 = 0.053$ m.

For three layers ($N = 3$) we get the system of three PDEs

$$\begin{cases} D_1(T)\dfrac{\partial^2 T_1(x,t)}{\partial x^2} = \dfrac{\partial T_1(x,t)}{\partial t}, \\ D_2(T)\dfrac{\partial^2 T_2(x,t)}{\partial x^2} = \dfrac{\partial T_2(x,t)}{\partial t}, \\ D_3(T)\dfrac{\partial^2 T_3(x,t)}{\partial x^2} = \dfrac{\partial T_3(x,t)}{\partial t}, \end{cases} \tag{6}$$

where $D_i(T) = \dfrac{K_i(T)}{\rho_i c_{pi}(T)}$, $i = 1, 2, 3$ – thermal diffusion coefficients depending on T.

For the initial condition for $t = 0$ we give $T_1(x,0) = T_2(x,0) = T_0$, where $T_0 = 20$ ºC.

We use the following boundary and continuous conditions:

$$\begin{cases} \dfrac{\partial T_1(0,t)}{\partial x} = 0, T_3(L,t) = g(t), \\ T_1(x_1,t) = T_2(x_1,t), T_2(x_2,t) = T_3(x_2,t), \\ D_1(T)\dfrac{\partial T_1(x_1,t)}{\partial x} = D_2(T)\dfrac{\partial T_2(x_1,t)}{\partial x} \\ D_2(T)\dfrac{\partial T_1(x_2,t)}{\partial x} = D_3(T)\dfrac{\partial T_2(x_2,t)}{\partial x}, \end{cases} \tag{7}$$

where $g(t) = 20 + 345\cdot\lg(8t + 1)$ ºC,
t – time in minutes.

Results and discussion

4. Some numerical results

The calculation results are obtained by MATLAB. The following discrete values are used:

$$x_j = j\cdot h,\ j = \overline{0, N_x},\ N_x\cdot h = L,\ t_n = n\cdot\tau,\ n = \overline{0, N_t},\ N_t\cdot\tau = t_f,$$

$$N_t = 100,\ t_f = 5000\,\text{s} = 83.3\,\text{min},\ t \in [0, t_f].$$

4.1. One layer gypsum material

We use the discrete values $N_x = 50$, $L = 0.025$ m. For nonlinear problem the calculation results with “pdepe” for $\rho = 1000$ kg·m^{-3} (gypsum board plate) are represented in Fig. 1, Fig. 3. The temperature values on the material layer boundaries at the end time t_f are $T(0,t_f) = 272.3$ ºC, $T(L,t_f) = 994.5$ ºC.

The results of the calculations with “pdepe” for $\rho = 300$ kg·m^{-3} (foam gypsum plate) are shown in Fig. 2, Fig. 4.

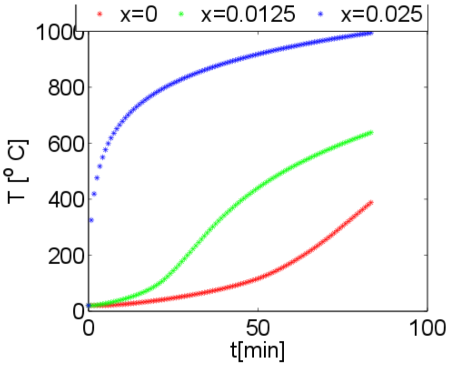

Fig. 1. **Temperature depending on *t* for *ρ* = 1000**

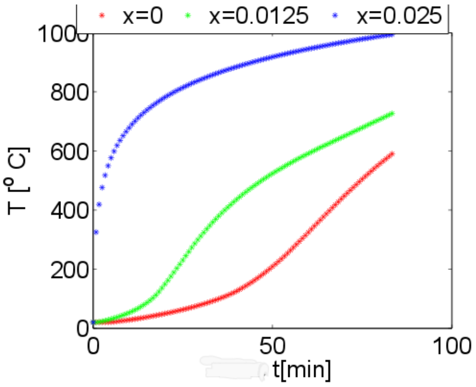

Fig. 2. **Temperature depending on *t* for *ρ* = 300**

In this case, the temperature values on the material layer boundaries at the end of the modelling time t_f are $T(0,t_f) = 589.9$ °C, $T(L,t_f) = 994.5$ °C, respectively. It is apparent that the temperature on the layer's boundary $x = 0$ at the end time t_f is higher than in the previous case ($\rho = 1000$ kg·m^{-3} $T(0,t_f) = 272.3$ °C), which indicates a lower heat resistance for the foam gypsum in comparison to the gypsum board in the process under consideration.

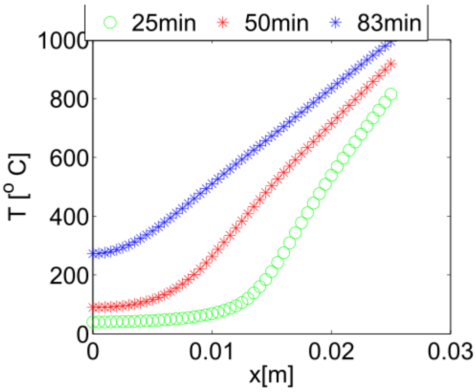

Fig. 3. **Temperature depending on *x* for ρ = 1000, *t* = 25; 50; 83 min**

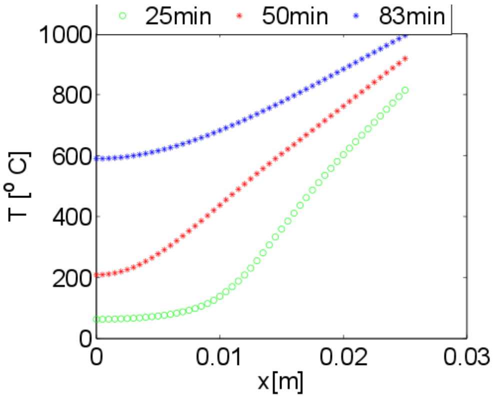

Fig. 4. **Temperature depending on *x* for ρ = 300, *t* = 25; 50; 83 min**

4.2. Two layer gypsum material

We use the values $N_x = 50$, $L = 0.050$ m. The results of the calculation with "pdepe" for $\rho_1 = 300$ kg·m^{-3} (foam gypsum plate), $\rho_2 = 1000$ kg·m^{-3} (gypsum board plate heated on the right) are shown in Fig. 5, Fig. 6 (the first case studied — "foam gypsum- gypsum board"), whereas the results of the calculation for $\rho_1 = 1000$ kg·m^{-3} (gypsum board plate), $\rho_2 = 300$ kg·m^{-3} (foam gypsum plate heated from the right) are illustrated in Fig. 7, Fig. 8 (the second case studied: "gypsum board - foam gypsum").

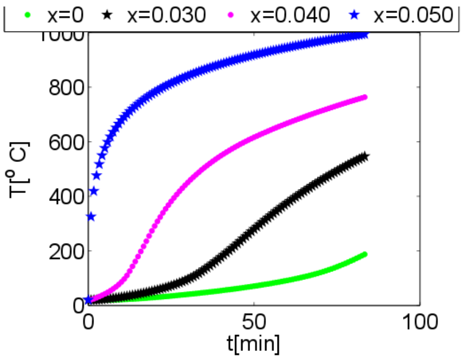

Fig. 5. **Temperature depending on *t* for fixed *x*, ρ_1 = 300, ρ_2 = 1000**

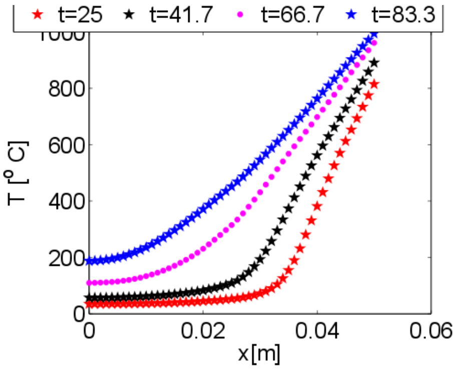

Fig. 6. **Temperature depending on *x* for fixed *t*, ρ_1 = 300, ρ_2 = 1000**

The temperature values on the boundaries of the layered material at the end time t_f are $T(0,t_f) = 187.5$ °C, $T(L,t_f) = 994.5$ °C, respectively (the case "foam gypsum – gypsum board"), and in the second case ("gypsum board - foam gypsum") t_f are $T(0,t_f) = 111.0$ °C, $T(L,t_f) = 994.5$ °C.

When viewing the temperature curves at the same x values, the gypsum board material appears to have a higher heat resistance, with a lower temperature compared to the foam gypsum material (see Fig. 6 $x < 0.025$ – foam gypsum material, $x \geq 0.025$ – gypsum board material; Fig. 8 $x < 0.025$ – gypsum board material, $x \geq 0.025$– foam gypsum material).

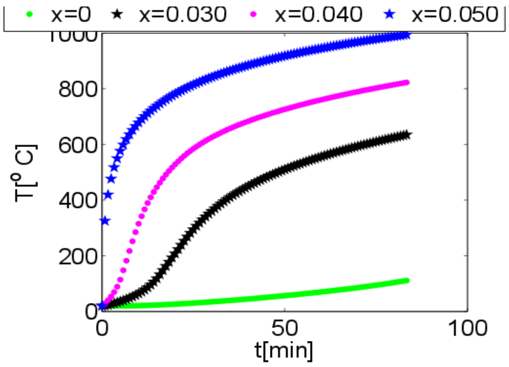

Fig. 7. **Temperature depending on t for fixed x, $\rho_1 = 1000$, $\rho_2 = 300$**

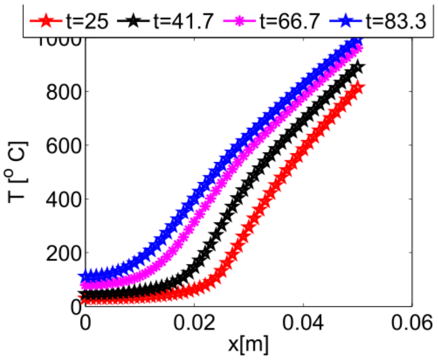

Fig. 8. **Temperature depending on x for fixed t, $\rho_1 = 1000$, $\rho_2 = 300$**

4.3. Three layer gypsum material

We use the values $N_x = 53$, $L = 0.053$ m. The maximal temperature 994.5 °C is obtained in the time t_f by $x = L = 0.053$ m. The results of the calculations with "pdepe" computer program for three-layered material "gypsum board – foam gypsum-gypsum board" are shown in Fig. 9 – Fig. 12. The temperature values at the characteristic points of the material have been calculated – $T(0,t_f) = 240.7$ °C (left boundary $x = 0$), $T(L,t_f) = 994.5$ °C (right boundary $x = L$), and at the individual internal points of the material – $T(0.016,t_f) = 466.1$ °C (in the second layer – "foam gypsum"), $T(0.040,t_f) = 813.6$ °C (in the second layer, "foam gypsum"). Comparing the given material to a two-layer material shows that the temperature at the left boundary of the given material at 240.7 °C is higher than for "foam gypsum- gypsum board" (187.5 °C), and also higher than for "gypsum board – foam gypsum" (111.0 °C).

The temperature distribution obtained corresponds to the results previously obtained for the higher heat resistance of the gypsum board material compared to the foam gypsum material.

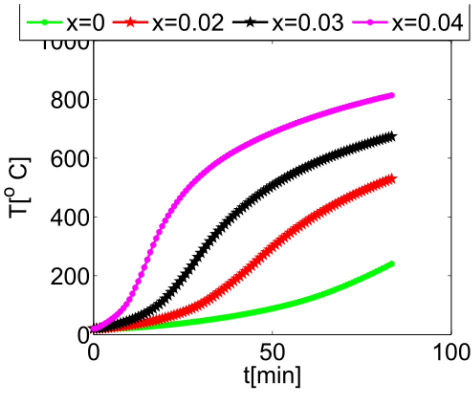

Fig. 9. **Temperature depending on t for fixed x**

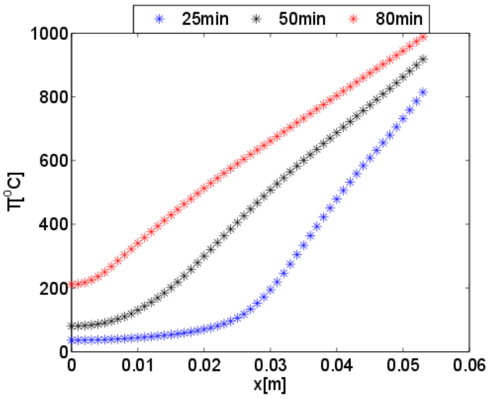

Fig. 10. **Temperature depending on x for fixed t = 25, 50, 80 min**

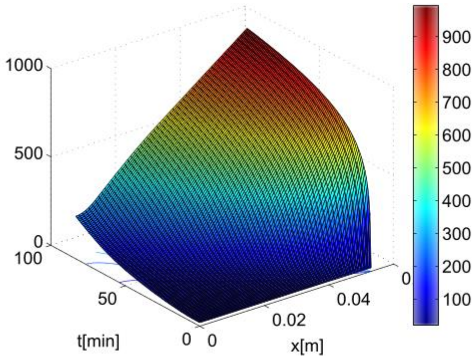

Fig. 11. **Surface of temperature depending on x, t**

Fig. 12. **Equipment for determining temperature dependency on x at fixed time t**

5. Some numerical and experimental results

5.1. Two layered gypsum board products

In the previous chapters (Chapter 4.2, Chapter 4.3) numerical experiments were conducted to find the temperature distribution in multiple layers of materials (foam gypsum and gypsum board) during their heating process.

The numerically obtained distributions of temperature were compared with the experimental measurement data in the burning-heating process of the above-mentioned gypsum and gypsum cardboard plates obtained at the Latvian University of the Life Sciences and Technologies (Faculty of Environment and Civil Engineering).

Comparison of the numerical results (*) and experimental measurement data (∇) on the left boundary ($x = 0$ m – the reverse side of the gypsum plasterboard burnt) of two-layer material is shown in Fig. 13 (“foam gypsum – gypsum board”) and Fig. 14 (“gypsum board– foam gypsum”).

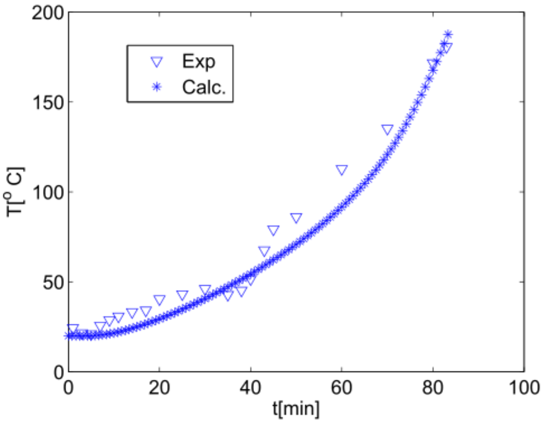

Fig. 13. **Comparison of the numerical and experimental data for temperature at $x = 0$, for $\rho_1 = 300$, $\rho_2 = 1000$ (material “foam gypsum- gypsum board”)**

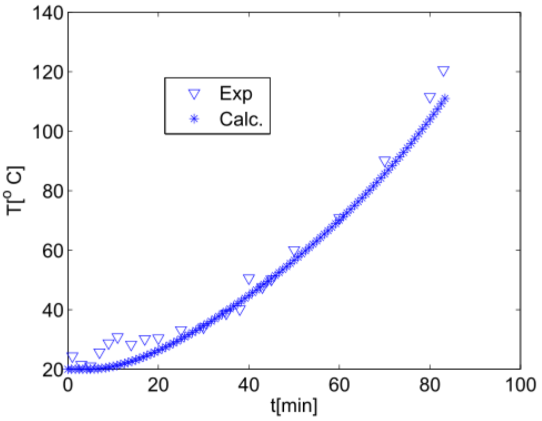

Fig. 14. **Comparison of the numerical and experimental data for temperature at $x = 0$, for $\rho_1 = 1000$, $\rho_2 = 300$ (material “gypsum board – foam gypsum”)**

5.2. Three layered gypsum board products

The values of numerical experiments are obtained (have been calculated) in the following points $x = 0$ m, $x = 0.016$ m, $x = 0.040$ m, $x = 0.053$ m of the studied three-layered material (“gypsum board-foam gypsum- gypsum board”).

In Fig. 15 there are compared the numerically calculated temperature distribution values and the experimental measurement data obtained in the burning-heating process depending on the time $t \in [0{,}0.83$ min] for the fixed $x = 0.053$; 0.040; 0.016; 0.0 m values (* – numerically calculated, ∇ – experimental measurement data).

The sequence of the graphs shown in Fig. 15 according to the values $x = 0.053$; 0.040; 0.016; 0.0 m must be looked at from top to bottom.

Fig. 13-15 show good coincidence of the experimental measurements with the obtained numerical calculations.

Different heat resistance of foam gypsum and gypsum board materials, in turn, affects the temperature distribution of the various foam gypsum and gypsum board materials considered.

For example, the temperature in the three-layered material (“gypsum board-foam gypsum-gypsum board”) at thickness $x = 0.040$ m is $T(0.040,t_f) = 813.6$ ºC (see chapter 4.3), whereas the temperature of the two-layered material (“gypsum board-foam gypsum”) is $T(0.040,t_f) = 822.9$ ºC and for the material “foam gypsum-gypsum board” is $T(0.040,t_f) = 763.6$ ºC.

The temperature values at x = 0.040 m are shown to be relatively lower in cases where a layer of gypsum board with higher heat resistance is located on the right side of the material where the heating with the heat source is carried out.

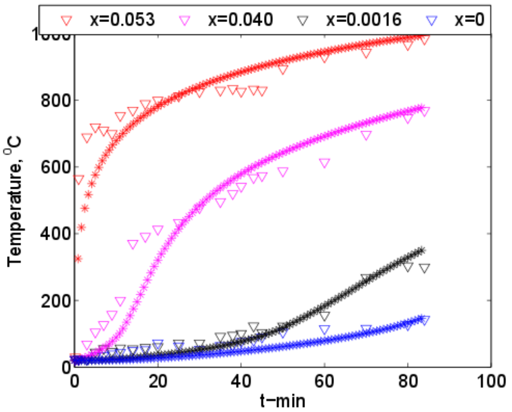

Fig. 15. **Comparison of the numerical and experimental data for temperature:**
* – numerically calculated;
∇ – experimental measurement data

Fig. 16. E**quipment and the result of the 3-layer material combustion experiment**

Conclusions

1. The mathematical model and the solution algorithm used, taking into account the temperature dependence of the specific heat and thermal conductivity coefficients, describe well the heat transfer in a layered gypsum medium in the event of a fire.
2. Gypsum board material, placed in the outer part in relation to the heat source (second case studied: (“gypsum board - foam gypsum”), is at a lower temperature than the gypsum material in the outer part (first case studied: “foam gypsum - gypsum board”) under the same conditions of heating and the same thickness. Consequently, the temperature value of the two-layered material “gypsum board- foam gypsum” is also lower at the left side x = 0 m of the material than in the first case.
3. The temperature value of the three-layered material “gypsum board -foam gypsum – gypsum board” at the left boundary in the heating process (at the end of the modelling process t = 83 min) was $T(0,t_f)$ = 240.7 ºC, it was higher than in the case of the two-layered material “gypsum board – foam gypsum” – $T(0,t_f)$ = 111.0 ºC.
4. This research indicates that when using foam gypsum in the partitions of dwellings, it would be better to cover it with thicker gypsum boards (more that 0.65 cm) based on fire safety conditions.

Author contributions

Conceptualization, A.A., I.K. and H.K.; methodology, H.K.; software, H.K. and I.K.; validation, A.A. and I.K.; formal analysis, A.A., I.K. and H.K.; investigation, A.A. and H.K.; data curation, A.A.; writing – original draft preparation, H.K.; writing – review and editing, I.K. and A.A.; visualization, H.K. and I.K.; funding acquisition, I.K.

All authors have read and agreed to the published version of the manuscript.

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