MATHEMATICAL MODEL OF MOVEMENT AND CLEANING BEETROOTS FROM SOIL LUMPS WITH SPIRAL SEPARATOR

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Abstract. An urgent problem in the technological process of table beet production is cleaning of the root crops from adhering soil, soil impurities and plant residues after digging them out of the soil. Therefore, it is necessary to develop new, more advanced designs of beet heap separators that could provide not only high quality cleaning but also eliminate damage and loss of the root crops when performing this technological process. A new design of a spiral separator has been developed, which, based on the results of preliminary experimental studies, satisfies the above requirements. Therefore, it is necessary to conduct fundamental theoretical investigations to substantiate the rational parameters of the specified separator. The purpose of this research is to determine the design and kinematic parameters of an improved design of the spiral separator for the table beet roots, ensuring high-quality cleaning of the root crops from the soil impurities and plant residues, based on the development of a mathematical model of the movement of the root crops along the working surface of the separator. An equivalent diagram of the interaction of the beet root as a material particle, located in the working channel, formed by three cantilever-mounted spirals, has been constructed. The separator spirals are driven into rotational motion and, at the same time, can oscillate in a vertical plane under the impact of the variable mass of the beet heap, supplied to the working surface. Based on an equivalent circuit, a system of differential equations for the movement of the beet root along the surface of the working channel has been compiled. The solution of the resulting system of differential equations makes it possible to study the influence of the design and kinematic parameters of the separator on the speed of movement of the root crop in the working channel before it leaves the spirals. By determining the design and kinematic parameters in this way it will ensure improved quality of cleaning the beet roots from the soil impurities and plant residues.

Keywords: beet, root crop, spiral separator, equivalent diagram, mathematical model, design and kinematic parameters.

Introduction

Table beetroot plays an important role in agriculture and human nutrition around the world. Due to its high content of vitamins, minerals and antioxidants, it is a valuable source of nutrients. Beets are also known for their benefits for heart health, digestion and the immune system. Its use in cooking is varied: from salads and soups to juices and canned food. In addition, beets are used in the food industry for the production of sugar and dyes. An important aspect is also its ability to be stored for a long time, which makes it available all year round. In agriculture beets are also used as livestock feed which complements its value in human and animal nutrition [1].

A serious agricultural problem is the soil loss during beet harvesting. During this process soil erosion occurs due to the mechanical impact of the agricultural machinery and manipulation of the root crops. The main causes of soil runoff include the use of heavy farm equipment, mismanagement and improper use of harvesting techniques. This may lead to decreased soil fertility, the loss of soil cover and environmental pollution. In order to reduce the soil losses, it is necessary to use modern harvesting methods and technologies, as well as to properly plan and organize the process to preserve the soil layer and its fertility for a long time [2-4]. Therefore, there is constant development and testing of new machines for harvesting and cleaning root beets [5; 6]. In works [7; 8] there is also noted the impact of the method of cleaning beets from the soil impurities upon the quality indicators and storage.

Important element of increasing the main indicators of the technological process of harvesting beets is cleaning them from the soil and plant residues immediately after digging them out of the soil [9].

One of the most important phases in the said work process is the cleaning of beet roots from soil and plant residues after their digging out from the soil [10-16]. This phase provides for the high quality of the harvested product, while the possibility of damaging the product in it must be ruled out [17].

To solve some problems in red beet harvesting, we can use a spiral separator as a prototype, which has shown good results [18; 19]. It delivers, according to the results of the implemented experimental investigations, a sufficiently high level of cleaning technological material from soil impurities and plant residues and at the same time ensures not damaging the tubers in terms of the established agricultural technology requirements. The simple design together with the low metal and energy intensity make this heap separator a promising engineering solution.

However, the varying conditions of harvesting, including the working environment condition (the humidity and hardness of the soil in the ridge area, the presence of stones and hardened soil bodies, haulm residues) as well as the quantity of technological material differing in their weights, dimensions and shapes, necessitate repeatedly changing the engineering and process characteristics of the above-mentioned separator in order to ensure the achievement of the described performance of the work process of cleaning [20].

Such a situation gives rise to a problem of scientific substantiation of such rational design and kinematic parameters of the spiral separator under consideration that would provide for the high quality of cleaning under different physical and mechanical properties of the soil and beet sizes. Therefore, carrying out fundamental theoretical research into the process of interaction between red beet roots and the operating components of the spiral separator under consideration is an important step towards solving the above-mentioned problem [21]. An example of a mathematical model of the interaction of technological material and separator spirals can be seen in paper [22]. It discusses the process of cleaning potato tubers from soil and plant residues with the use of the spiral separator subject to the requirement of not damaging the potato tubers. But the separation process of a heap of table beets will differ from that of potatoes, which will require the use of new technical solutions and additional theoretical research.

In order to study the process of cleaning beets from the soil and plant residues, we have developed a new spiral separator design (Fig. 1), which has certain advantages, compared to the existing separators.

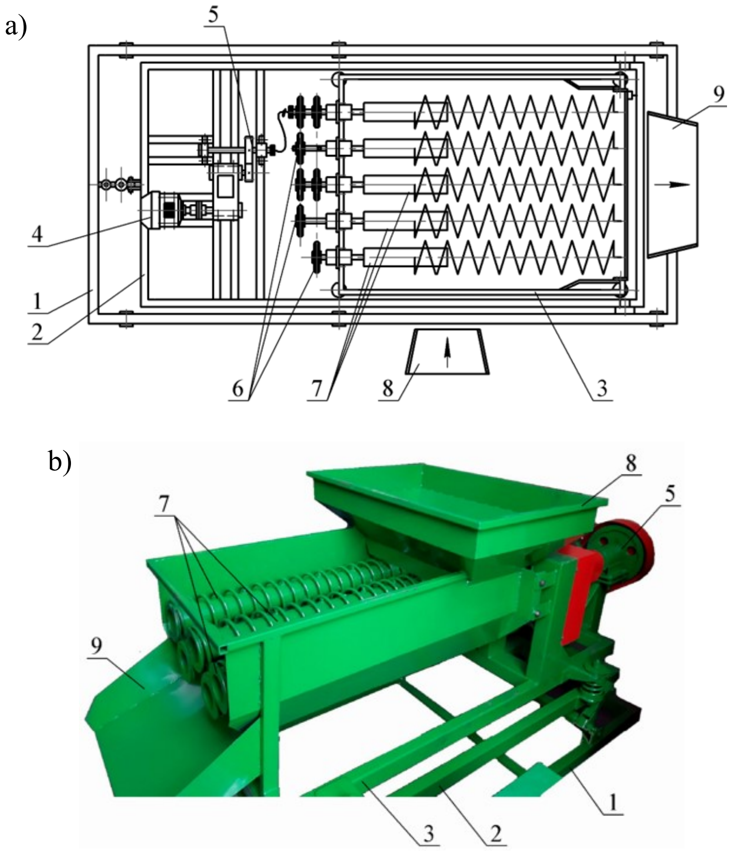

Fig. 1. **Structural scheme of the laboratory installation for the study of the process of cleaning beets from soil impurities (a), overall type of the laboratory installation (b):** 1 – fixed frame; 2 – swinging frame; 3 – vibro-frame; 4 – electric motor; 5 – reducer; 6 –drive of working tools; 7 – separating spirals; 8 – feeding tray; 9 – unloading tray

However, to select rational design and kinematic parameters of our proposed separator design (Fig. 1), it is necessary to conduct a series of theoretical and experimental investigations of interaction of the beets with the working bodies of the indicated separator. In terms of theoretical research, it is necessary to build a computational mathematical model of the interaction of the beet with the spiral separating surface of the design that we proposed. The developed mathematical model will make it possible to carry out numerical modelling, as a result of which the process under consideration will be comprehensively studied, and the rational design and kinematic parameters of the separator will be theoretically determined, taking into account non-damage to the beets during the cleaning process.

The purpose of the research is to determine the design and kinematic parameters of the improved design of a spiral separator for table beet roots, ensuring high-quality cleaning of the root crops from the soil impurities and plant residues, based on the development of a mathematical model of the movement of the root crops along the working surface of the separator.

Materials and methods

To construct a mathematical model of the movement of the table beetroot along the spirals of the separator of the proposed design, we identify the mentioned root in the form of a body, which is approximated as a cylinder with two hemispheres at its ends. The head and tail of the root crop are modelled by hemispheres. The tail of the root crop of indicated shape is so small, compared to the size of the root crop itself, that its existence can be neglected.

Diagram of interaction between table beetroots and separator spirals

Let us analyse the pattern of relative movement of the root crop along the separator spiral. As shown in Fig. 2, the working channel along which the root crop can move is formed by three spirals 1, 2 and 3 of the same radius *R*. In addition, spirals 2 and 3 are placed at a certain distance from each other. The longitudinal axes of these spirals are parallel to each other and located in a horizontal plane. Below, under them, spiral 1 is located, so that, as already mentioned, a working channel is formed. All three spirals are installed in a cantilever and rotate with the same angular velocity ω in the same direction (counter clockwise). Thus, one of the ends of these spirals is mounted on fixed drive shafts, but their other end is located freely. All three spirals have the same pitch *S* and the same helix angle γ.

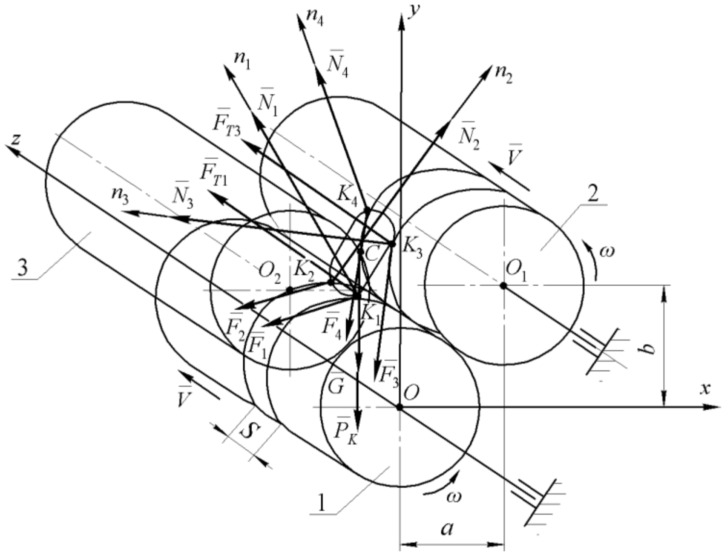

Fig. 2. **Equivalent diagram of interaction of table beetroot with separator spirals**

Let us describe one of the most likely options for the location of the root crop in the working channel. Initially the body of the root crop, designated by letter *C*, with one of its ends (the head or tail part) falls on spiral 2 or 3 (without the loss of generality, we will assume that it falls on spiral 2). When spiral 2 rotates and the gravity force of the root crop acts, after a very short time its falls with the other end on the lower spiral 1, thus contacting the surfaces of the spirals at four points K_1, K_2, K_3 and K_4, with the contact points K_1 and K_2 being located on spiral 1 on its adjacent turns, but the contact points K_3 and K_4 are on helix 2 on its adjacent turns.

The rotation of spirals 1 and 2 in the same direction causes the root crop to move in the said working channel in the longitudinal axial direction until it leaves these spirals. Since in our case the root crop is shaped more like a cylinder and is clearly not a ball, the rotational movement of the root crop around an axis, parallel to the longitudinal axes of the spiral springs, will not be characteristic of it.

Most likely, the coils of spiral springs 1 and 2 at the points of contact with the root crop will slide along its surface, thus involving the root crop discussed in forward motion along the working channel until it leaves this channel at the free end of the springs. This can be explained by the fact that the root crop, being in the cavity between spirals 1 and 2, is simultaneously located between two adjacent turns of spirals 1 and 2. These turns, when spirals 1 and 2 rotate, slide along the surface of the root crop, but holding it on both sides in the groove, formed by them, will move the root crop along the longitudinal axes of the spirals. Besides that, the cleaning of the root crop from the adhered soil will be partially carried out due to sliding of the spiral turns over the surface of the root crop, and the cleaning from the free soil and plant residues will take place by separating impurities through the inter-turn space of the spiral springs, due to the transverse vibrations of the cantilever springs in the vertical plane. Of course, these oscillations will be random in nature, but it is precisely they will contribute to the separation of the heap and the release of the surface of root crops from the adhered soil. However, the main movement of the root crop will be its forward movement along the longitudinal axes of the spirals. In addition, it should be noted that the above process of moving the root crops along the depression between the spirals in the axial direction is possible only at a certain angular speed of rotation of the spiral springs. When a certain critical angular velocity of rotation of the spirals is reached, the root crop may break away from the surface of the spiral and fly over the depression between the spirals, but such a value of the angular velocity of rotation of the springs should not be allowed. Moreover, this can lead to damage to root crops.

Since the main cleaning of the beet roots from adhered soil and sifting of the free soil and other impurities occurs during the movement of the root crop along the working channel between the spirals, it is necessary to study the movement of the root crop along the specified channel under the action of helical turns of the spirals, using the geometric properties of the helix.

To construct a computational mathematical model of the movement of the root crop in the cavity between the spirals, it is necessary to develop an equivalent diagram of its interaction with the surfaces of the separator spiral turns (Fig. 2). As shown in the diagram, the root crop is located in the cavity between helices 1 and 2, and its ends are located between two adjacent turns of both helix 1 and helix 2. Besides, the first turn in the direction of the movement of the root crop will be pushing, and the second will be supporting for both spirals. In addition, if under the impact of random factor the root crop in the working channel moves from spiral 2 to spiral 3, then the process will proceed similarly, but the contact of the root crop will already occur with spirals 1 and 3. If under the impact of random factors the root crop in the working channel moves from spiral 2 to spiral 3, then the process will proceed similarly, but the contact of the root crop will already occur with spirals 1 and 3.

As shown in the equivalent diagram (Fig. 2), at each contact point K_i, $i = (1, 2, 3, 4)$ normal reactions $\overline{N}_i$ ($i = 1, 2, 3, 4$).

The normal reaction $\overline{N}_i$, ($i = 1, 2, 3, 4$) is directed along the normal to the surface of the corresponding turns at the contact points K_i, ($i = 1, 2, 3, 4$). The direction of action of the normal reactions is formed by turns of spirals, the geometric properties of which are described by the equations of the helical line of each spiral. The aforementioned equations of the helical line of each spiral serve as coupling equations that specify the geometric properties of the trajectory of the movement of contact points K_i, ($i = 1, 2, 3, 4$) when they move along the turns of the spirals.

At the centre of mass of the root crop (point C) its force of gravity $\overline{G}$ is applied, which is directed vertically downward.

In addition to the forces already mentioned, the equivalent diagram shows the friction forces $\overline{F}_i$, ($i = 1, 2, 3, 4$), applied at the corresponding contact points K_i, ($i = 1, 2, 3, 4$). These forces arise when spiral turns 1 and 2 slip over the surface of the root crop. They are directed in the direction of rotation of the spirals, tangential to the surface of the coils at the points of their contact K_i with the surface of the root crop.

And finally, to ensure the movement of the root crop in the direction of the longitudinal axes of the spirals, the driving forces $\overline{F}_{T1}$ and $\overline{F}_{T3}$, applied at the contact points K_1 and K_3, respectively, act. These forces are directed parallel to the longitudinal axes of the spirals in the direction of the movement of the root crop in the working channel.

As it is known, the friction forces are related to the values of normal reactions by the following expressions:

$$F_i = f \cdot N_i, \ (i = 1, 2, 3, 4), \tag{1}$$

where f – coefficient of sliding friction of the root crop body along the surface of the spiral turns; for table beet this coefficient can be taken within the limits $f = 0.2 \ldots 0.3$ [23].

The force of gravity G of table beet is determined from the expression:

$$G = m \cdot g, \tag{2}$$

where m – mass (weight) of the root crop, kg;
g – acceleration of gravity, $m \cdot s^{-2}$.

The equivalent diagram also shows the active force $\overline{P}_K$ of the heap with the root crops supplied to the spiral separator, which leads to bending of the spirals and, due to its variable value because of changes in the mass of the supplied heap, causes transverse vibrations of the spirals. It is directed vertically downwards.

Mathematical model of moving table beets along the separator spirals

To compile differential equations for the movement of the table beet in the working channel, formed by the separator spirals, we choose a fixed spatial Cartesian coordinate system $xOyz$. The origin of which (point O) is located on the longitudinal axis of spiral 1, the Oz axis coincides with the longitudinal axis of spiral 1, the Oy axis is directed vertically upward, the Ox axis is directed to the right, perpendicular to the Oyz plane, and the Ox and Oy axes are located in the cross-sectional plane of spiral 1 (Fig. 2).

Taking into account the diagram of forces, shown in the equivalent diagram, using the basic law of dynamics of a material point, we write the equation of movement of the table beet in a vector form:

$$m \cdot \overline{a} = \overline{G} + \overline{N}_1 + \overline{N}_2 + \overline{N}_3 + \overline{N}_4 + \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4 + \overline{F}_{T1} + \overline{F}_{T3} + \overline{P}_K, \tag{3}$$

where $\overline{a}$ – acceleration of the movement of the table beet along the working channel under the impact of the forces indicated in the diagram, $m \cdot s^{-2}$.

Let us write the vector equation (3) in projections on the axes of the Cartesian coordinate system $xOyz$:

$$\left.\begin{aligned} m \cdot \ddot{x} &= \sum_{i=1}^{4} N_i \cdot \cos\left(x, ^\wedge \overline{n}_i\right) - \sum_{i=1}^{4} F_i \cdot \cos\left(x, ^\wedge \overline{V}_i\right), \\ m \cdot \ddot{y} &= \sum_{i=1}^{4} N_i \cdot \cos\left(y, ^\wedge \overline{n}_i\right) - \sum_{i=1}^{4} F_i \cdot \cos\left(y, ^\wedge \overline{V}_i\right) - G - P_K, \\ m \cdot \ddot{z} &= \sum_{i=1}^{4} N_i \cdot \cos\left(z, ^\wedge \overline{n}_i\right) - \sum_{i=1}^{4} F_i \cdot \cos\left(z, ^\wedge \overline{V}_i\right) + F_{T1} + F_{T3}, \end{aligned}\right\} \tag{4}$$

where $\overline{n}_i$ – normal to the surface of the helix at the point of contact K_i, (i = 1, 2, 3, 4);
$\overline{V}_i$ – velocity vector of relative movement of the table beet along the spiral turn at the point of contact K_i, (i = 1, 2, 3, 4), which is directed tangentially to the surface of the coil in the direction opposite to the peripheral speed of the coil at the point of contact K_i, (i = 1, 2, 3, 4), $m \cdot s^{-1}$.

Let us next determine the direction cosines of the angles, included in the system of differential equations (4). The direction cosines of the angles between the axes of the coordinate system $xOyz$ and the direction of the normals to the surface of the turns at the contact points K_i, (i = 1, 2, 3, 4), are determined according to the following expressions [24]:

$$\begin{gathered}\cos\left(x,\wedge\overline{n}_i\right)=\frac{\partial f_k}{\partial x}\cdot\left(\Delta f_k\right)^{-1},\\ \cos\left(y,\wedge\overline{n}_i\right)=\frac{\partial f_k}{\partial y}\cdot\left(\Delta f_k\right)^{-1},\\ \cos\left(z,\wedge\overline{n}_i\right)=\frac{\partial f_k}{\partial z}\cdot\left(\Delta f_k\right)^{-1},\\ \left(i=1,2,3,4\right),\quad\left(k=1,\ 2\right),\end{gathered}\tag{5}$$

where Δf_k – modulus of the function gradient $f_k(x, y, z)$, which is determined from the expression:

$$\Delta f_k=\sqrt{\left(\frac{\partial f_k}{\partial x}\right)^2+\left(\frac{\partial f_k}{\partial y}\right)^2+\left(\frac{\partial f_k}{\partial z}\right)^2},\tag{6}$$

$f_k(x, y, z)$ – coupling equation, which in this case is the equation of the spiral surface, $(k = 1, 2)$.

According to [23], for a cylindrical spiral 1 with the indicated dimensions (Fig. 2), the longitudinal axis of which coincides with the coordinate axis *Oz*, the coupling equation in the Cartesian coordinate system *xOyz* has the following form:

$$f_1=\frac{S^2}{4\pi^2}\cdot\left[\frac{x\cdot\sin\frac{2\pi z}{S}-y\cdot\cos\frac{2\pi z}{S}}{\sqrt{x^2+y^2}}\right]\cdot\cos\left(\frac{S}{2\pi\sqrt{x^2+y^2}}\right)+\left(\sqrt{x^2+y^2}-R\right)^2-r^2=0,\tag{7}$$

where R – radius of the spiral, m;
r – radius of the coil, m;
S – pitch of the helix, m.

Since the longitudinal axis of helix 2 is shifted to the right along the *Ox* axis relative to the axis of helix 1 by a distance, equal to a, and upward along the *Oy* axis by a distance, equal to b, its coupling equation in the *xOyz* coordinate system will have the following form:

$$\begin{gathered}f_2=\frac{S^2}{4\pi^2}\cdot\left[\frac{\left(x-a\right)\cdot\sin\frac{2\pi z}{S}-\left(y-b\right)\cdot\cos\frac{2\pi z}{S}}{\sqrt{\left(x-a\right)^2+\left(y-b\right)^2}}\right]\cdot\cos\left(\frac{S}{2\pi\sqrt{\left(x-a\right)^2+\left(y-b\right)^2}}\right)+\\ +\left(\sqrt{\left(x-a\right)^2+\left(y-b\right)^2}-R\right)^2-r^2=0.\end{gathered}\tag{8}$$

Taking into account expressions (5), it is necessary to determine the partial derivatives and gradient of the coupling functions. Differentiating (7) and (8) with respect to the variables x, y and z, we obtain expressions for the determination of the necessary partial derivatives and gradient of the coupling functions in the Cartesian coordinate system *xOyz*, which are conveniently presented in a parametric form. To do this, we write down the parametric equations of the helical lines (turns) of spirals 1 and 2 [24].

Parametric equation of helix 1:

$$\left.\begin{aligned}x&=R\cdot\cos\omega t;\\ y&=R\cdot\sin\omega t;\\ z&=-S\cdot\omega\cdot t\cdot\left(2\pi\right)^{-1},\end{aligned}\right\}\tag{9}$$

Due to the displacement of the longitudinal axis of the spiral 2 to the right by the distance a and to the top by the distance b, its parametric equation will have the form:

$$\left.\begin{array}{l} x-a=-R\cdot\cos\omega t; \\ y-b=-R\cdot\sin\omega t; \\ z=-S\cdot\omega\cdot t\cdot\left(2\pi\right)^{-1}. \end{array}\right\} \qquad (10)$$

In equations (9), (10): R– radius of the spiral, $\psi = \omega t$ – independent angular parameter of the spiral, which specifies the placement of the cross section along the length of the spiral.

Substituting the parametric equations of helical lines (9) and (10) into the expressions for determination of the necessary partial derivatives and gradient of the coupling functions in the Cartesian coordinate system, after a series of transformations we obtain a representation of the indicated partial derivatives and gradient in parametric form.

Taking into account expressions (5), we obtain the values of the direction cosines of the angles between the coordinate axes Ox, Oy and Oz and the normal reactions of the spiral turns at the points of contact K_i, (i = 1, 2, 3, 4) with the surfaces of the spiral turns 1 and 2 in a parametric form:

$$\begin{aligned}&\cos\left(x,\hat{}\,\bar{n}_1\right)=\cos\left(x,\hat{}\,\bar{n}_2\right)=-\cos\left(x,\hat{}\,\bar{n}_3\right)=-\cos\left(x,\hat{}\,\bar{n}_4\right)=\\&=\left(L\cdot\sin\omega t+M\cdot\cos\omega t\cdot\sin 2\omega t\right)\times\\&\times\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1},\end{aligned} \qquad (11)$$

$$\begin{aligned}&\cos\left(y,\hat{}\,\bar{n}_1\right)=\cos\left(y,\hat{}\,\bar{n}_2\right)=\cos\left(y,\hat{}\,\bar{n}_3\right)=\cos\left(y,\hat{}\,\bar{n}_4\right)=\\&=\left(L\cdot\cos\omega t+M\cdot\sin\omega t\cdot\sin 2\omega t\right)\times\\&\times\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1},\end{aligned} \qquad (12)$$

$$\begin{aligned}&\cos\left(z,\hat{}\,\bar{n}_1\right)=-\cos\left(z,\hat{}\,\bar{n}_2\right)=\cos\left(z,\hat{}\,\bar{n}_3\right)=-\cos\left(z,\hat{}\,\bar{n}_4\right)=\\&=Q\cos 2\omega t\cdot\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1}.\end{aligned} \qquad (13)$$

where

$$L=-\frac{S^2}{4\pi^2 R}\cdot\cos\left(\frac{S}{2\pi R}\right), \qquad (14)$$

$$M=\frac{S^2}{4\pi^2 R}\cdot\cos\left(\frac{S}{2\pi R}\right)-\frac{S^3}{8\pi^3 R^2}\cdot\sin\left(\frac{S}{2\pi R}\right), \qquad (15)$$

$$Q=\frac{S}{2\pi}\cdot\cos\left(\frac{S}{2\pi R}\right). \qquad (16)$$

Next we will determine the direction cosines of the friction forces.

This means determination of the cosines of the angles between the vectors of relative speeds of movement of the root crop along the turns of the spirals at the points of contact K_i, (i = 1, 2, 3, 4), and the coordinate axes Ox, Oy and Oz, that is, $\cos(x,\hat{}\,\bar{V}_i)$, $\cos(y,\hat{}\,\bar{V}_i)$ and $\cos(z,\hat{}\,\bar{V}_i)$, ($i$ = 1, 2, 3, 4).

Besides, it should be taken into account that vectors $\bar{V}_1$ and $\bar{V}_2$ are both collinear, and vectors $\bar{V}_3$ and $\bar{V}_4$ are also collinear. Therefore, the following equalities hold:

$$\cos\left(x,\hat{}\,\bar{V}_2\right)=\cos\left(x,\hat{}\,\bar{V}_1\right),\ \cos\left(y,\hat{}\,\bar{V}_2\right)=\cos\left(y,\hat{}\,\bar{V}_1\right),\ \cos\left(z,\hat{}\,\bar{V}_2\right)=\cos\left(z,\hat{}\,\bar{V}_1\right), \qquad (17)$$

$$\cos\left(x,\hat{}\,\bar{V}_4\right)=\cos\left(x,\hat{}\,\bar{V}_3\right),\ \cos\left(y,\hat{}\,\bar{V}_4\right)=\cos\left(y,\hat{}\,\bar{V}_3\right),\ \cos\left(z,\hat{}\,\bar{V}_4\right)=\cos\left(z,\hat{}\,\bar{V}_3\right). \qquad (18)$$

Direction cosines $\cos(x,\hat{}\,\bar{V}_i)$, $\cos(y,\hat{}\,\bar{V}_i)$ and $\cos(z,\hat{}\,\bar{V}_i)$, ($i$ = 1, 2, 3, 4), in the Cartesian coordinate system can be found, using the following expressions [24]:

$$\cos\left(x,\hat{}\bar{V}_i\right)=\frac{\dot{x}}{V_i}=\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}, \tag{19}$$

$$\cos\left(y,\hat{}\bar{V}_i\right)=\frac{\dot{y}}{V_i}=\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}, \tag{20}$$

$$\cos\left(z,\hat{}\bar{V}_i\right)=\frac{\dot{z}}{V_i}=\frac{\dot{z}}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}. \tag{21}$$

Substituting into (19)-(21) derivatives with respect to time t from expressions (9)-(10), we obtain the necessary values of the cosines of the angles between the vectors $\bar{V}_i$, (i = 1, 2, 3, 4), and the coordinate axes Ox, Oy and Oz in a parametric form:

$$\cos\left(x,\hat{}\bar{V}_2\right)=\cos\left(x,\hat{}\bar{V}_1\right)=\frac{2\pi R\cdot\sin\omega t}{\sqrt{4\pi^2R^2+S^2}}, \tag{22}$$

$$\cos\left(x,\hat{}\bar{V}_4\right)=\cos\left(x,\hat{}\bar{V}_3\right)=-\frac{2\pi R\cdot\sin\omega t}{\sqrt{4\pi^2R^2+S^2}}, \tag{23}$$

$$\cos\left(y,\hat{}\bar{V}_2\right)=\cos\left(y,\hat{}\bar{V}_1\right)=-\frac{2\pi R\cdot\cos\omega t}{\sqrt{4\pi^2R^2+S^2}}, \tag{24}$$

$$\cos\left(y,\hat{}\bar{V}_4\right)=\cos\left(y,\hat{}\bar{V}_3\right)=\frac{2\pi R\cdot\cos\omega t}{\sqrt{4\pi^2R^2+S^2}}, \tag{25}$$

$$\cos\left(z,\hat{}\bar{V}_2\right)=\cos\left(z,\hat{}\bar{V}_1\right)=\cos\left(z,\hat{}\bar{V}_4\right)=\cos\left(z,\hat{}\bar{V}_3\right)=\frac{S}{\sqrt{4\pi^2R^2+S^2}}. \tag{26}$$

In this case the magnitude of the velocity vector of the relative movement of the root crop along the turns of the spirals will be equal to:

$$V_i=\frac{\omega}{2\pi}\cdot\sqrt{4\pi^2R^2+S^2},\ \left(i\ =\ 1,\ 2,\ 3,\ 4\right). \tag{27}$$

Substituting expressions (11) -(13) and (22)-(26) into the system of differential equations (4), after a series of transformations we obtain the following system of differential equations in a parametric form:

$$\left.\begin{aligned}
&m\cdot\ddot{x}=\left(N_1+N_2-N_3-N_4\right)\cdot\left(L\cdot\sin\omega t+M\cdot\cos\omega t\cdot\sin 2\omega t\right)\times\\
&\times\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1}-\\
&-\left(F_1+F_2-F_3-F_4\right)\cdot\frac{2\pi R\cdot\sin\omega t}{\sqrt{4\pi^2R^2+S^2}},\\
&m\cdot\ddot{y}=\left(N_1+N_2+N_3+N_4\right)\cdot\left(L\cdot\cos\omega t+M\cdot\sin\omega t\cdot\sin 2\omega t\right)\times\\
&\times\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1}-\\
&-\left(-F_1-F_2+F_3+F_4\right)\cdot\frac{2\pi R\cdot\cos\omega t}{\sqrt{4\pi^2R^2+S^2}}-mg-P_K,\\
&m\cdot\ddot{z}=\left(N_1-N_2+N_3-N_4\right)\cdot Q\cos 2\omega t\times\\
&\times\left(\sqrt{L^2+\left(2ML+M^2\right)\cdot\sin^2 2\omega t+Q^2\cos^2 2\omega t}\right)^{-1}-\\
&-\left(F_1+F_2+F_3+F_4\right)\cdot\frac{S}{\sqrt{4\pi^2R^2+S^2}}+F_{T1}+F_{T3},
\end{aligned}\right\} \tag{28}$$

The system of differential equations (28) is a system of differential equations for the movement of the table beet under the action of turns of the rotating spirals along the working channel, formed by cantilever spiral springs, where the root crop is cleaned of the soil impurities and plant residues.

Parametric recording of the resulting system of differential equations allows to set the initial conditions (at $t = 0$). Indeed, from the system of equations (9) we obtain the values of the coordinates of the contact points K_1 and K_2 at the initial moment of time $t = 0$:

point K_1 ($x_0 = R$, $y_0 = 0$, $z_0 = 0$); point K_2 ($x_0 = R$, $y_0 = 0$, $z_0 = S$).

From the system of equations (10) we obtain the coordinate values of the contact points K_3 and K_4 at the initial moment oftime $t = 0$:

Point K_3: ($x_0 = a - R$, $y_0 = b$, $z_0 = 0$); point K_4 ($x_0 = a - R$, $y_0 = b$, $z_0 = S$).

The values of the initial velocities of the contact points K_1 and K_2 are obtained from equations (9) by differentiating them: $\dot{x}_0 = 0$, $\dot{y}_0 = -\omega R$, $z_0 = \dfrac{S\omega}{2\pi}$.

The values of the initial velocities of the contact points K_3 and K_4 are obtained from equations (10) by differentiating them: $\dot{x}_0 = 0$, $\dot{y}_0 = \omega R$, $z_0 = \dfrac{S\omega}{2\pi}$.

As a result of double integration of system (28), taking into account the initial conditions, it is possible to obtain the laws of movement of the table beet along the working channel of the spiral separator, as a function of time t and the values of its parameters.

However, system (28) includes unknown normal reactions $\bar{N}_i$ ($i = 1, 2, 3, 4$) which can be defined as follows. Since in a steady state the rotation of the spirals takes place with constant angular velocities ω = const, then with such a steady motion the root crop will move with a constant linear speed $\bar{V}_i$ ($i = 1, 2, 3, 4$), relative to the surface of the spiral turns, defined by expression (27). Therefore, to a first approximation in the absolute coordinate system $xOyz$, the projections of the speed of movement of the root crop in the working channel of the separator on the axis of the mentioned coordinate system, taking into account the slippage of the turns along the surface of the root crop, can be considered constant.

Since we took the projections of the speed of the root crop on the coordinate axes to be constant, the projections of the acceleration of the root crop on the axes Ox, Oy and Oz can be considered equal to zero, that is, $\ddot{x} = 0$, $\ddot{y} = 0$, $\ddot{z} = 0$. Therefore, the left-hand sides of the system of differential equations (28) become zero, as a result of which this system turns into a system of linear algebraic equations for unknowns $\bar{N}_i$ ($i = 1, 2, 3, 4$), with variable coefficients.

For convenience and brevity of writing this system of equations, we introduce the following notation:

$$\begin{aligned}&\left(L \cdot \sin\omega t + M \cdot \cos\omega t \cdot \sin 2\omega t\right)\times\\&\times\left(\sqrt{L^2 + \left(2ML + M^2\right)\cdot\sin^2 2\omega t + Q^2\cos^2 2\omega t}\right)^{-1} = M_1\end{aligned}, \tag{29}$$

$$\begin{aligned}&\left(L \cdot \cos\omega t + M \cdot \sin\omega t \cdot \sin 2\omega t\right)\times\\&\times\left(\sqrt{L^2 + \left(2ML + M^2\right)\cdot\sin^2 2\omega t + Q^2\cos^2 2\omega t}\right)^{-1} = M_2\end{aligned}, \tag{30}$$

$$Q\cos 2\omega t \cdot \left(\sqrt{L^2 + \left(2ML + M^2\right)\cdot\sin^2 2\omega t + Q^2\cos^2 2\omega t}\right)^{-1} = M_3, \tag{31}$$

$$\frac{2\pi R \cdot \sin\omega t}{\sqrt{4\pi^2 R^2 + S^2}} = L_1, \tag{32}$$

$$\frac{2\pi R \cdot \cos\omega t}{\sqrt{4\pi^2 R^2 + S^2}} = L_2, \tag{33}$$

$$\frac{S}{\sqrt{4\pi^2R^2+S^2}}=L_3. \tag{34}$$

Taking into account expression (1) and notations (29)-(34), the system of linear algebraic equations will be written in the following form:

$$\left.\begin{aligned}&(N_1+N_2-N_3-N_4)\cdot M_1-f\cdot(N_1+N_2-N_3-N_4)\cdot L_1=0,\\&(N_1+N_2+N_3+N_4)\cdot M_2-f\cdot(-N_1-N_2+N_3+N_4)\cdot L_2=mg+P_K,\\&(N_1-N_2+N_3-N_4)\cdot M_3-f\cdot(N_1+N_2+N_3+N_4)\cdot L_3=-(F_{T1}+F_{T3}).\end{aligned}\right\} \tag{35}$$

Since there are three equations in system (35) and four unknowns, it does not have a unique solution. Therefore, let us assume as a first approximation, for reasons of symmetry (Fig. 2), that $N_1 = N_3$, $N_2 = N_4$. Then the system of equations (35) will take the following form:

$$\left.\begin{aligned}&2(N_1+N_2)\cdot M_2=mg+P_K,\\&2(N_1-N_2)\cdot M_3-2f\cdot(N_1+N_2)\cdot L_3=-(F_{T1}+F_{T3}).\end{aligned}\right\} \tag{36}$$

Solving the system of equations (36) for the unknowns N_1 and N_2, we obtain:

$$N_1=\frac{(mg+P_K)\cdot(M_3+f\cdot L_3)-(F_{T1}+F_{T3})\cdot M_2}{4M_2M_3}=N_3, \tag{37}$$

$$N_2=\frac{(mg+P_K)\cdot(M_3-f\cdot L_3)+(F_{T1}+F_{T3})\cdot M_2}{4M_2M_3}=N_4. \tag{38}$$

Thus, with sufficient for practice accuracy, the normal reactions $\bar{N}_i (i = 1, 2, 3, 4)$ of turns of spirals 1 and 2 at the points of contact with the surface of the table beet have been determined.

It is further necessary to determine the driving forces F_{T1} and F_{T3}, which are included in the system of differential equations (28).

Results and discussion

We have carried out experimental and theoretical investigations in order to determine the driving forces F_{T1} and F_{T3} that ensure the movement of table beets in the longitudinal direction of the spirals.

As the experiments have shown, the power $P_{\Pi P}$ to drive the spiral springs (3 pieces) under a working load can reach a value of 850 W at the angular speed ω of rotation of the spiral springs close to 40 rad·s^{-1} and the feed of the heap mass of up to 30 kg·s^{-1}. At the same time it was experimentally determined that the maximum value of power consumed by only one spiral to create a driving force F_{Ti}, ($i = 1, 3$) is, on average, 85 W.

Based on the value of the power P_{1S} consumption, we can determine the torque M_K on the drive shaft of one spiral using the following relationship:

$$M_K=\frac{P_{1S}}{\omega}, \tag{39}$$

where M_K – torque on the drive shaft of one spiral, N·m^{-1};
ω – angular speed of rotation of the spiral, rad·s^{-1}.

Taking into account (39), we find the value of the driving force F_T, created by one spiral:

$$F_{Ti}=\frac{M_K}{R}\cdot\sin\gamma=\frac{P_{1S}}{\omega\cdot R}\cdot\sin\gamma,\ (i\ =\ 1,\ 3), \tag{40}$$

where R – radius of the spiral, m;
γ – angle of elevation of the helix of the spiral, deg.

Also, based on the results of experimental investigations, the value of the force P_K, included in the system of differential equations (28) was determined. Experiments have shown that the maximum mass of the soil lumps, located in the heap, arriving on the surface of the separator, is in most cases approximately equal to the average mass of the table beets. Therefore, based on the results of the measurements, we assume that the force P_K of action of the supplied heap on an individual root crop is equal to mg, that is:

$$P_K = mg, \tag{41}$$

where m – mass of table beets, kg.

Thus, all the necessary values of the forces, included in the system of equations (28), have been determined.

To carry out a numerical calculation of the resulting mathematical model, it is necessary to set the kinematic and design parameters of the separator.

Let us model the movement parameters of the table beets, using the mathematical apparatus that we have developed.

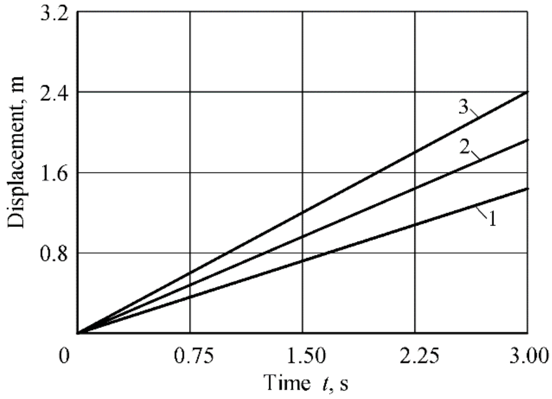

Fig. 3. **Dependence of the movement of table beets along the channel of spirals depending on time *t* at:** 1 – ω = 30 rad·s^{-1}; 2 – ω = 40 rad·s^{-1}; 3 – ω = 50 rad·s^{-1}

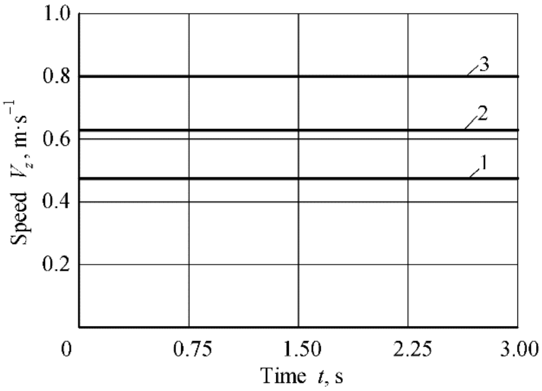

Fig. 4. **Dependence of the speed of table beets along the channel of spirals as a function of time *t* at:** 1 – ω = 30 rad·s^{-1}; 2 – ω = 40 rad·s^{-1}; 3 – ω = 50 rad·s^{-1}

Analysing the obtained dependencies (Fig. 3 and Fig. 4), we see that, due to the contact of the beet root with the separator spirals at four points, it is given the speed of movement of the spirals themselves, and an increase in the rotation speed and pitch of the spirals leads to a proportional increase in the speed of the root itself. Also, based on the obtained dependencies (Fig. 3), we can determine the required time for the heap remaining on the separating surface if we specify the design and kinematic parameters of the separator (the spring length and pitch, rotation speed).

Conclusions

1. A mathematical model has been constructed for the movement of table beets along the working channel of a spiral separator, formed by three spiral springs, on the basis of which it is possible to calculate rational design parameters and kinematic operating modes of the specified separator, using the calculation method.

2. As a result of calculations, graphical dependences of the impact of the design and kinematic parameters of the separator upon the process of cleaning table beets from adhered soil, soil impurities and plant residues were obtained.
3. The solution of the resulting mathematical model allows to obtain a relationship between the design and kinematic parameters of the separator (the spring length and pitch, rotation speed) and the movement parameters of the table beets. For example, the time t the table beet root crop leaves the separating surface, which is 1 meter long, is 1.25 s, 1.56 s, and 2.09 s at spiral rotation frequencies ω of 30, 40, and 50 rad·s^{-1}, respectively. This is with a spiral winding step of S = 0.09 m, and the speed V_z of the root crop descent from the separating springs will be 0.47 m·s^{-1}, 0.63 m·s^{-1} and 0.8 m·s^{-1}, respectively.

Author contributions

Conceptualization, V.B., I.H. and V.M.; methodology, V.B., I.H., A.R. and A.A.; software, Y.I.; validation, A.R. and O.T; formal analysis, V.B and V.M.; investigation, V.B., I.H., V.M. and O.T.; data curation, O.T., V.B. an Y.I.; writing original draft preparation, V.B.; writing review and editing, V.B., I.H. and Y.I.; visualization, Y.I., O.T.; project administration, V.B.; funding acquisition, V.B. All authors have read and agreed to the published version of the manuscript.

References

[1] Karklelienė R., Vikelis P., Radzevičius A., Duchovskienė L. Evaluation of productivity and biochemical composition of perspective red beet breeding number. Acta Horticulturae, 830. 2009, pp. 255-260. DOI: 10.17660/ActaHortic.2009.830.35.
[2] Tuğrul K.M., Altınay Perendeci E.I.N. Determination of soil loss by sugar beet harvesting. Soil and Tillage Research 23, 2012, pp. 71-77. DOI: 10.1016/j.still.2012.03.012.
[3] Ruysschaert G., Poesen J., Verstraeten G., Govers G. Inter annual variation of soil losses due to sugar beet harvesting in West Europe. Agriculture, Ecosystems Environment 107(4), 2004, pp. 317-329.
[4] Vargas-Ramirez Juan M., Haagenson Darrin M., Pryor Scott W., Wiesenborn Dennis P. Beet tissue ensiling: An alternative for long-term storage of sugars in industrial beets for nonfood use. Biomass and Bioenergy 85, 2016, pp. 135-143. DOI: 10.1016/j.biombioe.2015.12.003.
[5] Lammers S., Schmittmann O. Testing of sugar beet harvesters in Germany 2012. Int. Sugar J. 115, 2013, pp. 100-106.
[6] Ziegler K. Beet Europe Seligenstadt – Erntemaschinen im Test. DZZ 6., 2012, 24–25.
[7] Lammers S., Jürgen P., Jürgen S. Progress in soil tare separation in sugar beet harvest. Journal of Plant Nutrition and Soil Science 166, 2003, pp. 126-127. DOI: 10.1002/jpln.200390004.
[8] Hoffmann C., Engelhardt M., Gallmeier M., Gruber M., Märländer B. Importance of harvesting system and variety for storage losses of sugar beet. Zuckerindustrie. Sugar industry 143, 2018, 474. DOI: 10.36961/si19782.
[9] Ivanišová E., Granátová L., Čech M., Grygorieva O. Kubiak P. The influence of red beet varieties and cultivation on their biochemical and sensory profile. Journal of microbiology, biotechnology and food sciences. 2024 DOI: e10612. 10.55251/jmbfs.10612.
[10] Misener G.C., McLeod C.D. Resource efficient approach to potato-stone-clod separation. AMA, Agricultural Mechanization in Asia, Africa and Latin America 20(2), 1989, pp. 33-36.
[11] Peters R. Damage of potato tubers: A Review. Potato Research 39 (Spec. Issue), 1997, pp. 479-484.
[12] Veerman A. Wustman R. Present state and future prospects of potato storage technology. Potato in Progress: Science Meets Practice, Book Chapter, 2005, pp. 179-189.
[13] Bishop C., Rees D., Cheema M.U.A., Harper G. Stroud G. Potatoes. Crop PostHarvest: Science and Technology: Perishables. Book Chapter, 2012, pp. 179-189.
[14] Ichiki H., Nguyen Van N., Yoshinaga K. Stone-clod separation and its application to potato in Hokkaido. Engineering in AgricultureEnvironment and Food 6(2), 2013, pp. 77-85.
[15] Guo W., Campanella O.H. A relaxation model based on the application of fractional calculus for describing the viscoelastic behavior of potato tubers. Transactions of the ASABE 60(1), 2017, pp. 259-264.

[16] Wang X., Sun J., Xu Y., Li X., Cheng P. Design and experiment of potato cleaning and sorting machine. Nongye Jixie Xuebao/Transactions of the Chinese Society for Agricultural Machinery 48(10), 2017, pp. 316–322 and 279.
[17] Petrov G. Potato harvesting machines. Mashinostroenije, Moskow, 2004,320 pp (In Russian).
[18] Bulgakov V., Nikolaenko S., Adamchuk V., Ruzhylo Z., Olt J. Theory of retaining potato bodies during operation of spiral separator. Agronomy Research 16(1), 2018, pp. 41-51. DOI: 10.15159/AR.18.036
[19] Bulgakov V., Nikolaenko S., Adamchuk V., Ruzhylo Z., Olt J. Theory of impact interaction between potato bodies and rebounding conveyor. Agronomy Research 16(1), 2018, pp. 52-64. DOI: 10.15159/AR.18.037.
[20] Feller R., Margolin E., Hetzroni A., Galili N. Impingement angle and product interference effects on clod separation. Transactions of the American Society of Agricultural Engineers 30(2), 1987, pp. 357-360.
[21] Krause F. Minkin A. Research on shaftless screw conveyors. Bulk Solids Handling 25(2), 2005, pp. 92-100.
[22] Bulgakov V., Ivanovs S., Adamchuk V. Ihnatiev Ye. Investigation of the parameters of the experimental spiral potato heap separator on the quality of work. Agronomy Research 15(1), 2017, pp. 44-54.
[23] Bulgakov V., Nikolaenko S., Adamchuk V., Ruzhylo Z., Olt J. Mathematical model of cleaning potatoes on surface of spiral separator. Agronomy Research 16(4), 2018, pp. 1590-1606. DOI: 10.15159/AR.18.173.
[24] Vasilenko P.M. Introduction to agricultural mechanics. Kyiv, Agricultural Education, 1996, 252 p.