IDENTIFICATION PROCEDURE OF FUNCTIONALLY GRADED MATERIAL IN SANDWICH BEAM

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Abstract. The present work considers two types of functionally graded material with the through-the-thickness distributions in the core of a sandwich beam. A numerical study of free vibrations of a sandwich beam was carried out using the finite element method (using ANSYS) and corresponding modal parameters (natural frequencies and mode shapes) were determined. Deflection of a simply-supported sandwich beam was investigated additionally. The effects of material parameters were discussed in detail. Two models developed enable to determine a single material parameter using the frequency or displacement responses, if the remaining material data or distribution laws are available. By means of the model I, the inverse problem solution allows to calculate the values of material properties at the bottom surface of the core if the properties are linearly distributed through-the-thickness of the core material and are known at the top surface. Application of the model II allows to determine the law of distribution of material properties through-the-thickness of the core if material properties are known only on the top and bottom surfaces.

Keywords: numerical calculations, experiment, frequency, functionally graded material, sandwich, beam.

Introduction

In recent years, functional gradient materials (FGMs) as a new generation of modern composites have been increasingly applied in various branches of technology. FGMs are the materials with the physical or mechanical properties changing in the volume. FGMs have a combination of properties that differ from the original elements of structure and ensure the adaptation of the material to the conditions required for application. A gradual changing of the mechanical properties within the volume makes it possible to produce new materials with a wider range of applications compared to the traditional composites [1-3].

Smooth changing of the mechanical properties (modulus of elasticity, relaxation processes, hardness, etc.) could be realized in a prescribed direction of FGM. The traditional method of controlling the cross-sectional properties of laminates, including the sandwiches, is based on a formation of the structure with different mechanical properties in each layer. As a result, such laminates have a nonhomogeneous distribution of mechanical properties through the cross-section in contrast to FGMs with a prescribed law of distribution of mechanical properties over the thickness.

At present, the researchers are applying FGM in the sandwich structures as a filler. The filler is made of single or multiple materials, which allows one to reduce the weight of the structure, increase the strength, and improve the dynamic characteristics. Functionally graded polymer foams (FGPFs) can be used as a filler. These foams are a kind of porous polymers having a spatial gradient composition or structure [4; 5]. The application of syntactic foams is investigated now. These polymer matrix composites include hollow microspheres dispersed in the matrix. The matrices used in the syntactic foams may be polymer, metal or ceramic ones [6, 7]. Functionally graded cellular materials have recently been investigated. The main results showed that gradually increasing the cell thickness could improve the mechanical properties compared to homogeneous cell structures [8]. Functionally graded cellular cores of sandwich panels are fabricated by additive manufacturing technology.

The vibration analysis of functionally graded beam structures is the subject of detailed research. Wattanasakulpong [9] examined vibration analysis of functionally graded beams with porosity effects. Wang et al. [10] analyzed the forced vibration response of sandwich FG beams. The bending analysis of a porous FG beam using a two-node 4-DOF finite element is investigated by Zghal et al. [11].

The properties of a gradient material may change during the processes of its production or operation. Therefore, it is necessary to control the material properties using non-destructive identification methods. The problems of identification the properties of materials are relevant in many areas of science and technology. Knowledge of the mechanical characteristics of materials and understanding of their behavior allows the development and production of more efficient and durable structures. Therefore, identification properties of the gradient materials are important for their application. To solve such problems, it is necessary to develop new effective methods of non-destructive testing that are sufficiently

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precise and easy applicable in engineering practice. One of the experimental approaches to determining the elastic characteristics of composite materials is based on studying the vibrations of specimens. This approach is a non-destructive testing method and does not require expensive equipment. The development of computer modelling stimulated the development of a new method for determining the elastic characteristics of a composite material, based on solving the inverse problem [12]. The principle of the method is experimental and numerical determination of the natural frequencies and natural modes of vibrations of a specimen using the finite element method. Dynamic characteristics can be obtained by analyzing free vibrations [13] and natural frequencies [14]. By calculating the minimum discrepancy between the experimental and numerical values of natural frequencies, the elastic characteristics of the material are determined.

In the open literature, identification of FGM properties in a sandwich beam was not described. The main purpose of this work is preliminary numerical research of a sandwich beam with a functionally graded core for identification of the material properties in the core. The problem considered is solved by using the finite element method. FEM analysis of the free vibrations of porous FG sandwich beams with simple support is carried out. Natural frequencies and deflections of the FG sandwich beams were calculated and analyzed by means of parametric study for various material properties and the power law distribution.

Materials and methods

To carry out numerical studies, we considered a three-layered sandwich beam with the length L = 400 mm, width b = 50 mm, and the thickness h = 22 mm. The model under study consists of a filler with a thickness of h_c = 20 mm and two outer layers with a thickness of 1.0 mm each. The schematic structure of the beam is shown in Fig. 1. Load-bearing aluminum layers of the beam were used. To study the gradient properties of the core, the properties of Rohacell polymethacrylimide foam were varied.

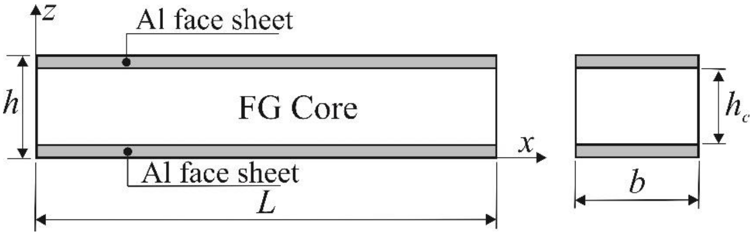

Fig. 1. **Geometry of sandwich beam**

The gradient distributions of the elastic $E(z)$ and shear $G(z)$ moduli, and the core density $\rho(z)$ in the core of the sandwich beam along the thickness (z-coordinate) are calculated using the following formulae [15]:

$$E(z) = (E_t - E_b)\left(0.5 + z/h_c\right)^p + E_b, \tag{1}$$

$$G(z) = (G_t - G_b)\left(0.5 + z/h_c\right)^p + G_b, \tag{2}$$

$$\rho(z) = (\rho_t - \rho_b)\left(0.5 + z/h_c\right)^p + \rho_b, \tag{3}$$

$$\nu = \frac{E(z)}{2G(z)} - 1 = \text{const.} \tag{4}$$

Here, the subscripts t and b designate the material properties on the top and bottom surfaces of the core, respectively. The value of Poisson's ratio ν is calculated by means of the elastic E and shear G moduli. Thus, Poisson's ratio ν is assumed as constant and equal to 0.25 according to Eq. (4). The index p used in Eqs. (1)-(3) describes the distribution of material properties through the thickness of each layer. The material properties considered in this study are as follow: E = 70 GPa, ρ = 2700 kg·m^{-3} (for the aluminium layers), ν = 0.3 and E = 35 MPa, ρ = 52 kg·m^{-3}, ν = 0.25 (for the core layer).

For the numerical study, two models of the distribution of gradient properties through the thickness of the core were considered.

Model I. The values of the elastic and shear moduli and density of the core material on its top surface are taken as the above-presented data. The properties of the core material on its bottom surface vary with changing the multiplier k in the range from 0.5 to 1.0. The index p is taken equal to unity, which corresponds to the linear distribution of material properties through the core thickness. Example of power law distribution of Young's modulus E through the non-dimensional thickness coordinate z/h of the core is presented in Fig. 2. It is seen that Young's modulus E at the top surface of the core is 35 MPa, whereas it varies from 17.5 to 35 MPa at the bottom surface of the core; these values correspond to the minimum and maximum values of the coefficient k equaled to 0.5 and 1.0, respectively.

Model II. The values of the elastic and shear moduli, as well as the density of the core material on its top and bottom surfaces, differ by a factor of two. Index p changes in the range from 0.1 to 10. An example of the power law distribution of Young's modulus E through the non-dimensional thickness coordinate z/h of the core is presented in Fig. 3 for nine values of the index p. It is seen that values of Young's modulus E = 35 and 17.5 MPa on the top and bottom surfaces of the core, respectively.

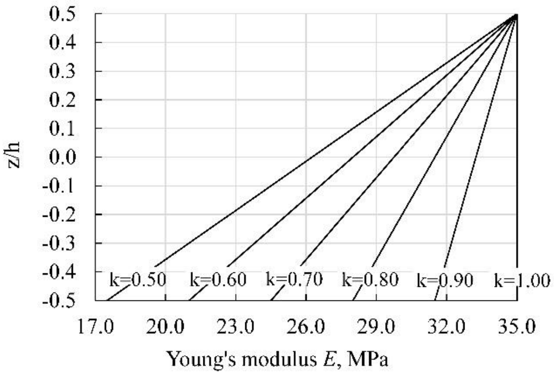

Fig. 2. **Power law distribution: Model 1**

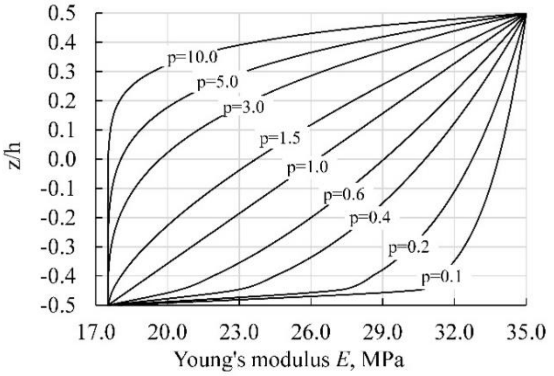

Fig. 3. **Power law distribution: Model 2**

To carry out a numerical experiment using the finite element method, 3D finite element (FE) model of a sandwich beam with a functionally graded core was built using ANSYS16.0 software. To simulate the core structure of the sandwich beam, 8-node SOLID185 solid elements were used. To simulate the top and bottom face skins, 4-node SHELL 181 shell elements were used. The FG core in the sandwich beam was modelled in ANSYS with a step-wise material gradation from the top of the core (E_t = 35 MPa) to bottom (E_b = 17.5-35.0 MPa) using 20 layers (1-mm thick each layer) across the beam thickness of 20 cm where each layer is assumed to be isotropic and homogenous based on the volume fraction. The ANSYS model and material distribution (Model 1) are shown in Fig. 4.

Before starting numerical calculations, the study of FE mesh convergence was carried out to obtain an acceptable accuracy. According to the results of the convergence study, we selected the solid elements of 1-mm thickness and 2-mm width.

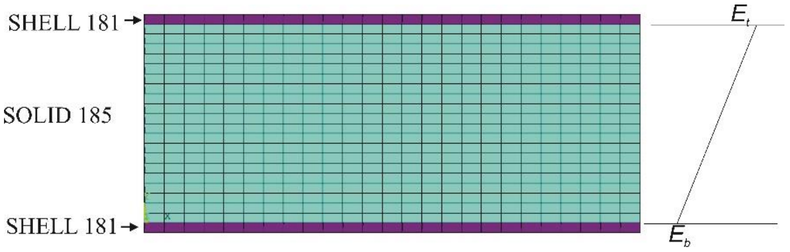

Fig. 4. **Cross section in finite element model of sandwich beam**

The determination of the natural frequencies and mode shapes of vibrations of the sandwich beam was performed using the ANSYS Modal subroutine and Mechanical APDL solver. First six natural frequencies and mode shapes were determined using free-free boundary conditions. The static load numerical tests are carried out additionally. Simply supported boundary conditions were used for modelling of the sandwich beam deflection (Fig. 5). The span between two supports is 400 mm. Bending load P is 400 N.

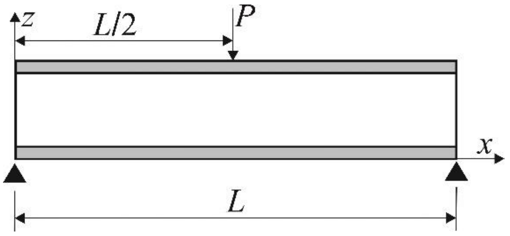

Fig. 5. **Schematic diagram of bending**

Results and discussion

At the first stage, the problem of determining the natural vibration frequencies and mode shapes of the sandwich beam with a functionally graded core was solved for the Model 1. Using the ANSYS software, six mode shapes and frequencies were determined in the range up to 1,400 Hz (Table 1). It can be seen that the natural frequencies of the sandwich beam with homogeneous structure ($k = 1$) decreases with reduction of the coefficient k. Thus, a decrease of the properties of the core material leads to decreasing the values of natural frequencies.

Table 1

Values of natural frequencies (in Hz) calculated using Model I

No.	**Mode shape**	**Coefficient k**					
		0.50	0.60	0.70	0.80	0.90	1.00
f_1	Twist 1	433.3	447.7	460.6	472.2	482.8	492.1
f_2	Bending 1	439.0	452.6	464.4	474.7	483.9	492.7
f_3	Bending 2	667.6	691.1	711.8	730.2	746.8	762.0
f_4	Bending 3	948.5	981.2	1010.1	1035.9	1059.4	1080.8
f_5	Twist 2	1164.5	1185.8	1204.8	1221.8	1237.2	1251.5
f_6	Bending 4	1189.9	1230.6	1266.9	1299.3	1329.0	1356.3

The dependence of the 1st natural frequency f_1 on the coefficient k is presented in Fig. 6. Thus, if the values of properties of the core material on its top surface are known, which are distributed linearly through-the-thickness ($p = 1$), then the inverse problem solution allows to calculate the values of the material properties at the bottom surface of the core. So, if we calculate a numerical model with a coefficient $k = 0.83$, which is not included in the range under study (see Table 1) and assume that the properties of the material are unknown, then from the obtained value of the 1st frequency $f_1 = 475.5$ Hz (see Fig. 6) we can determine the value multiplier k and, accordingly, the properties of the core material on its bottom surface. If it is not possible to determine the value of the 1st natural frequency experimentally, then values of other natural frequencies can be used to determine the properties of the core material on its bottom surface.

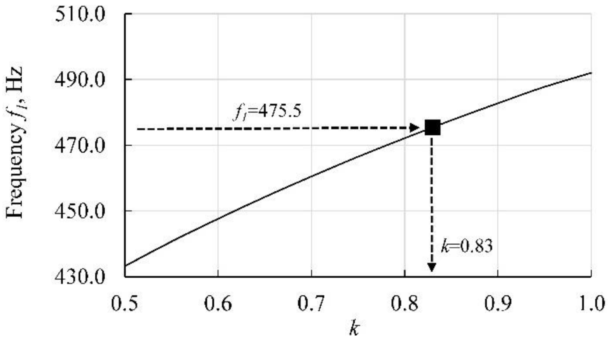

Fig. 6. **Dependence of frequency f_1 vs. k**

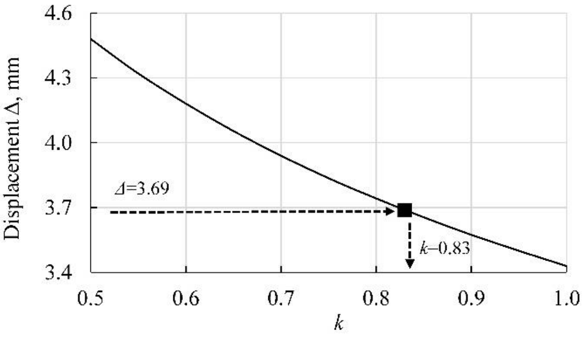

Fig. 7. **Dependence of displacement Δ vs. k**

The dependence of the sandwich beam displacement Δ on the coefficient k is presented in Fig. 7. It is seen that the beam displacement increases with decreasing the coefficient k. Increasing the beam displacement is explained by decreasing the stiffness of the material with decreasing the elastic modulus of the core. Solving the inverse problem makes it possible to predict the properties of the core material at its bottom surface by means of the displacements of the sandwich beam, using a diagram of the dependence of the coefficient *k* on the displacement Δ of the sandwich beam.

At the second stage, the problem of determining the natural vibration frequencies and mode shapes of the sandwich beam with a functionally graded core for the Model 2 was solved (Table 2). It was observed that an increase in the value of the index *p* leads to a reduction of the natural frequency. The highest values of natural frequencies are obtained for maximal values of material properties of the core ($p = 0.1$), while the lowest values of frequencies are obtained for minimal values of material properties of the core ($p = 10$). This is due to the fact that an increase in the value of the index *p* leads to decreasing the value of the elastic modulus. In other words, the beam becomes more flexible as the index *p* increases, which reduces the values of the natural frequencies.

Table 2

Values of natural frequencies (in Hz) calculated using Model II

No.	**Mode shape**	**Index *p***								
		0.1	0.2	0.4	0.6	1.0	1.5	3.0	5.0	10.0
f_1	Twist 1	476.5	469.5	457.7	447.9	433.3	421.4	403.8	394.3	386.1
f_2	Bending 1	478.0	471.9	461.4	452.5	439.0	427.8	411.4	402.4	394.6
f_3	Bending 2	736.4	725.4	706.7	691.1	667.6	648.7	620.9	605.9	593.0
f_4	Bending 3	1044.7	1029.2	1003.0	981.3	948.5	922.1	883.3	862.2	844.1
f_5	Twist 2	1227.3	1217.2	1200.1	1185.9	1164.5	1147.1	1108.2	1081.8	1059.2
f_6	Bending 4	1310.6	1291.1	1258.1	1230.9	1189.9	1156.8	1121.7	1107.8	1095.8

Solving the inverse problem, when the properties of the core material are known only on the top and bottom surfaces, makes it possible to determine the law of distribution of material properties through the thickness by means of the index *p*. So, if we calculate a numerical model with the index $p = 6.8$, which is not included in the range under study (see Table 2) and assume that the distribution of material properties is unknown, then from the obtained values of the 1st frequency $f_1 = 390.0$ Hz (Fig. 8) we can determine the index *p* and, accordingly, the law of distribution of material properties through the thickness of the core.

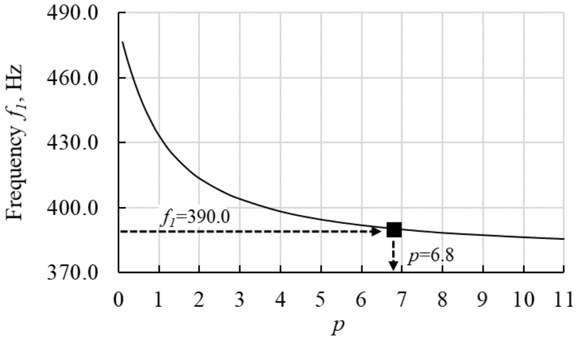

Fig. 8. **Dependence of frequency f_1 vs. *p***

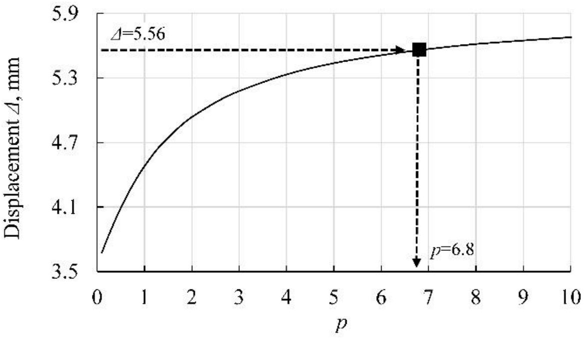

Fig. 9. **Dependence of displacement Δ vs. *p***

Figure 9 illustrates the effect of the index *p* on the deflection of the sandwich beams under load. It is seen that increasing the index *p* reduces the stiffness of the sandwich beam, and as result, leads to increasing its displacement. This is due to the fact that higher values of the index *p* correspond to the highest material properties at the bottom surface in comparison with the material properties at the top surface. As results, sandwich beam becomes more flexible. Solution of the inverse problem makes it possible to predict the distribution of material properties through-the-thickness of core by means of the displacements of the sandwich beam. Knowing the properties of the core on its top and bottom surfaces and using the displacement values allows to find the value of the index *p*, which determines the distribution law of the material through the thickness.

Conclusions

1. Two models of the sandwich beams with a functionally graded core were developed, computed, and extensively studied.

2. Highest natural frequency values of the sandwich beam using Model I were determined for maximal values of the material properties on the top and bottom surfaces of the core ($k = 1$), while the lowest frequency values were determined for minimal values of the material properties on the bottom surface ($k = 0.5$). Decrease of the coefficient k reduces the stiffness of the sandwich beam, and as result, leads to increasing its displacement.
3. The highest natural frequency values of the sandwich beam using Model II were determined for maximal values of the material properties of the core ($p = 0.1$) while the lowest frequency values are obtained for minimal values of the material properties of the core ($p = 10$). Increase of the index p reduces the stiffness of the sandwich beams, and as result, leads to increasing its displacement.
4. For each model developed, it is possible to determine one material parameter by means of the frequency response or displacement, if the remaining material data or distribution laws are available. However, the problem of determining the properties of the core material cannot be solved only through a parametric study if it is necessary to determine more than one parameter. Therefore, the development of the experimental plan and response surface to identify the properties of the gradient filler has to be continued.

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Author contributions

Conceptualization, E.B. and O.V.; methodology, E.B.; software, A.K.; validation A.K.; formal analysis, A.K. and O.V.; investigation, A.K.; data curation, writing – original draft preparation, A.K. and E.B.; writing – review and editing, A.K. and E.B.; visualization, A.K. and E.B.; project administration, E.B.; funding acquisition, E.B.; All authors have read and agreed to the published version of the manuscript.

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